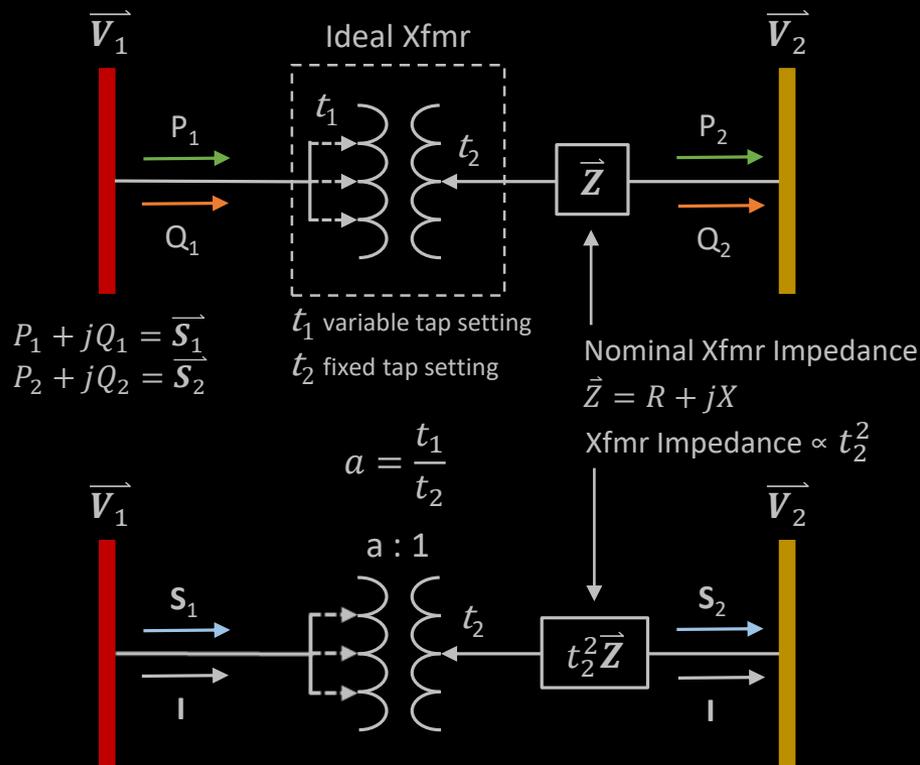


# Transformer Flow Considering Tap Changes

all values per-unit

$$t_{1 pu} = \frac{tap_1 \text{ setting}}{V_{1 nominal}}$$

$$t_{2 pu} = \frac{tap_2 \text{ setting}}{V_{2 nominal}}$$



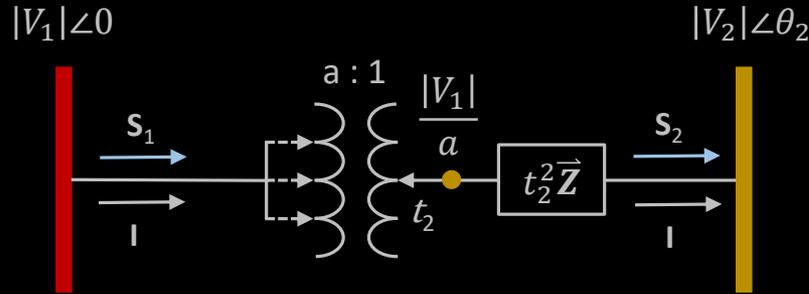
Choose  $V_1$  as reference and analyze side 2

Note:

PSSE does these calculations automatically.

The variable tap changer must be specified on side 1 (from bus).

# Transformer Flow Considering Tap Changes (cont.)



$$\vec{I} = \frac{\frac{|V_1|}{a} - |V_2| \angle \theta_2}{t_2^2 \vec{Z}} \quad \vec{S}_2 = \vec{I}^* |V_2| \angle \theta_2$$

$$\vec{I} = \frac{\vec{S}_2^*}{|V_2| \angle -\theta_2}$$

$$\frac{\frac{|V_1|}{a} - |V_2| \angle \theta_2}{t_2^2 \vec{Z}} = \frac{\vec{S}_2^*}{|V_2| \angle -\theta_2}$$

$$|V_2| \angle (-\theta_2) \left[ \frac{|V_1|}{a} - |V_2| \angle \theta_2 \right] = \vec{S}_2^* t_2^2 \vec{Z}$$

$$\frac{|V_1| |V_2| \angle (-\theta_2)}{a} - |V_2|^2 = t_2^2 (P_2 - jQ_2)(R + jX)$$

$$|V_1| |V_2| \angle (-\theta_2) - a |V_2|^2 = a t_2^2 (P_2 - jQ_2)(R + jX)$$

$$|V_1| |V_2| [\cos(-\theta_2) + j \sin(-\theta_2)] - a |V_2|^2 = a t_2^2 (P_2 - jQ_2)(R + jX)$$

$$|V_1| |V_2| [\cos(\theta_2) - j \sin(\theta_2)] - a |V_2|^2 = a t_2^2 (P_2 - jQ_2)(R + jX)$$

$$|V_1| |V_2| [\cos(\theta_2) - j \sin(\theta_2)] - a |V_2|^2 = a t_2^2 [(P_2 R + Q_2 X) + j(P_2 X - Q_2 R)]$$

$$|V_1| |V_2| \cos(\theta_2) - a |V_2|^2 - j |V_1| |V_2| \sin(\theta_2) = a t_2^2 (P_2 R + Q_2 X) + j a t_2^2 (P_2 X - Q_2 R)$$

*Equating Real and Imaginary terms:*

$$|V_1| |V_2| \cos(\theta_2) - a |V_2|^2 = a t_2^2 (P_2 R + Q_2 X)$$

$$-|V_1| |V_2| \sin(\theta_2) = a t_2^2 (P_2 X - Q_2 R)$$

# Transformer Flow Considering Tap Changes (cont.)

$$|V_1||V_2| \cos(\theta_2) - a|V_2|^2 = \underbrace{at_2^2(P_2R + Q_2X)}_B \quad -|V_1||V_2| \sin(\theta_2) = at_2^2(P_2X - Q_2R)$$

$$|V_1||V_2| \sin(\theta_2) = \underbrace{at_2^2(Q_2R - P_2X)}_C$$

since these are known... make a substitution

$$|V_1||V_2| \cos(\theta_2) - a|V_2|^2 = aB$$

$$|V_1||V_2| \sin(\theta_2) = aC$$

$$\cos(\theta_2) = \frac{aB + a|V_2|^2}{|V_1||V_2|}$$

$$\sin(\theta_2) = \frac{aC}{|V_1||V_2|}$$

$$\cos^2(\theta_2) = \frac{a^2B^2 + 2a^2B|V_2|^2 + a^2|V_2|^4}{|V_1|^2|V_2|^2}$$

$$\sin^2(\theta_2) = \frac{a^2C^2}{|V_1|^2|V_2|^2}$$

$$\cos^2(\theta_2) + \sin^2(\theta_2) = 1$$

$$\frac{a^2B^2 + 2a^2B|V_2|^2 + a^2|V_2|^4}{|V_1|^2|V_2|^2} + \frac{a^2C^2}{|V_1|^2|V_2|^2} = 1$$

$$a^2B^2 + 2a^2B|V_2|^2 + a^2|V_2|^4 + a^2C^2 = |V_1|^2|V_2|^2$$

$$a^2B^2 + 2a^2B|V_2|^2 + a^2|V_2|^4 + a^2C^2 - |V_1|^2|V_2|^2 = 0$$

$$a^2B^2 + 2a^2B|V_2|^2 + a^2|V_2|^4 + a^2C^2 - |V_1|^2|V_2|^2 = 0$$

$$a^2|V_2|^4 + 2a^2B|V_2|^2 - |V_1|^2|V_2|^2 + a^2B^2 + a^2C^2 = 0$$

$$a^2|V_2|^4 + (2a^2B - |V_1|^2)|V_2|^2 + a^2(B^2 + C^2) = 0$$

we ended up with a mess ...

- 4<sup>th</sup> order equation
- numerical solution required

After  $V_2$  is found ...

$$\theta_2 = \sin^{-1} \left( \frac{aC}{|V_1||V_2|} \right)$$

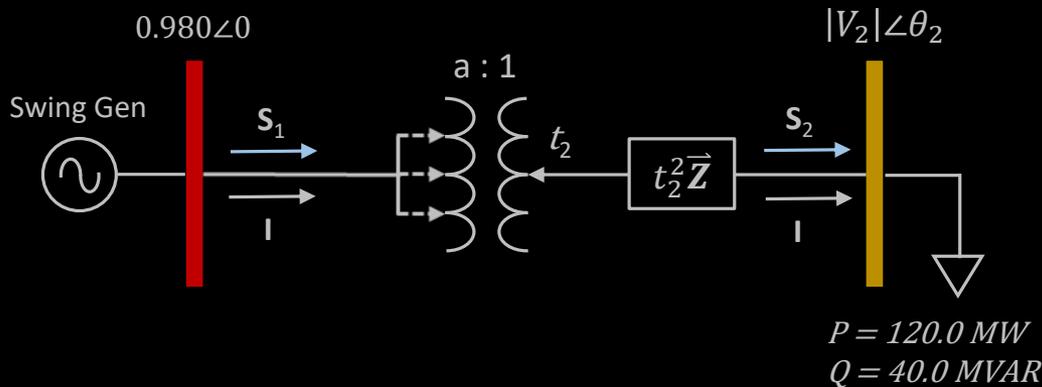
$$I = \frac{\frac{|V_1|}{a} - V_2 \angle \theta_2}{t_2^2(R + jX)}$$

$$S_1 = \frac{|V_1|}{a} I^*$$

# Transformer Flow Considering Tap Changes (cont.)

## Numerical Solution

let's put some numbers in and try to find a solution ...



Power flow simulations must have a swing generator ...

- maintains Bus1 voltage at  $0.980_{\text{pu}}$  (for this example) and  $0^\circ$
- provides real and reactive power needed to solve the system (load + losses)

Nominal voltage of Bus1 is 138kV

Nominal voltage of Bus2 is 69kV

$t_1$  is 0.9875 (136.275 kV) (step 2)

$t_2$  is 1.025 (70.725 kV)

$\therefore a = 0.9634$

Transformer impedance is 8.0% @200MVA with  $X/R = 30.0$

$\therefore Z = 0.00133 + j0.03998$  @ 100MVA

Load is  $1.20 + j0.40$  @100MVA

$\therefore S_2 = 1.20 + j0.40$

Find:  $V_2, I, S_1$

### Why is a numerical solution required?...

- this type of problem is known as load flow or power flow
- notice that the load is not a "passive" impedance
- if the load were simply  $R + jX$ ... simple solution
- since the load power is constant ...
- the load power will determine the current
- the current will determine the Bus2 voltage
- If Bus2 voltage increases ...
- current must decrease to provide the same power
- If current is changed ... Bus2 voltage will change
- so you have to iterate through Bus2 voltage values to find the operating point that satisfies the Constant load power

we will solve this one with a short python script that uses the Newton – Raphson method

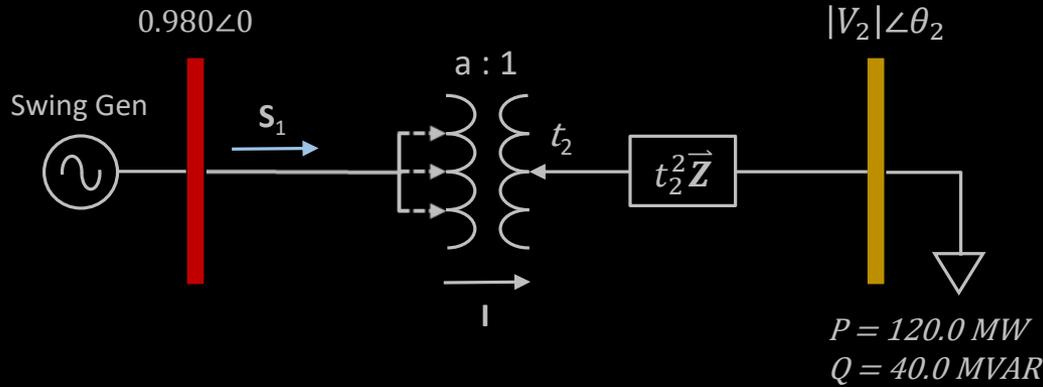
$$a^2|V_2|^4 + (2a^2B - |V_1|^2)|V_2|^2 + a^2(B^2 + C^2) = 0$$

# Transformer Flow Considering Tap Changes (cont.)

## Numerical Solution

$$B = P_2 R + Q_2 X$$

$$C = Q_2 R - P_2 X$$



$$a^2 |V_2|^4 + (2a^2 B - |V_1|^2) |V_2|^2 + a^2 (B^2 + C^2) = 0$$

Newton – Raphson requires the derivative:

$$4a^2 |V_2|^3 + 2(2a^2 B - |V_1|^2) |V_2|$$

each iteration adjusts the value of  $V_2$  until a solution is found

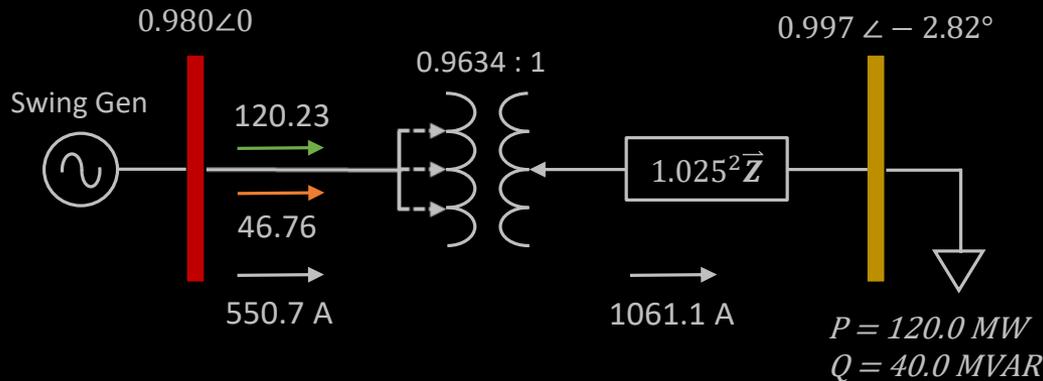
After  $V_2$  is found ...

$$\theta_2 = \sin^{-1} \left( \frac{aC}{|V_1| |V_2|} \right)$$

$$\mathbf{I} = \frac{\frac{|V_1|}{a} - V_2 \angle \theta_2}{t_2^2 (R + jX)}$$

$$\mathbf{S}_1 = \frac{|V_1|}{a} \mathbf{I}^*$$

Solution ...

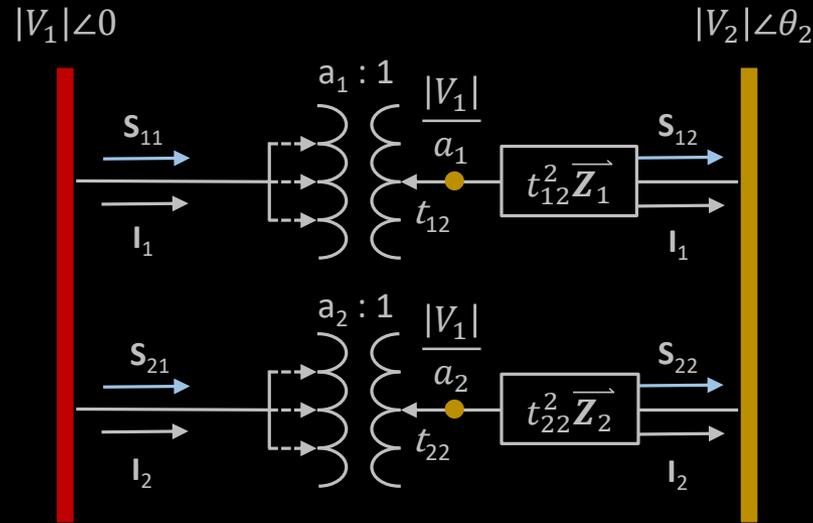


# Newton – Raphson solution

```
import math
def get_tap_flow_data(kv1,kv2,v1,t1,t2,r, x, p, q):
    v2 = 1.0
    a = t1 / t2
    z = complex(r, x)
    p /= 100.0; q /= 100.0
    b = (t2**2)*(p*r + q*x); c = (t2**2)*(q*r - p*x)
    iterations = 0; tol = 1e-5; max_iterations = 99
    f_v2 = (a**2)*(v2**4) + (2*a**2*b - v1**2)*(v2**2) + (a**2)*((b**2)+(c**2))
    while abs(f_v2) > tol and iterations < max_iterations:
        f_v2 = (a**2)*(v2**4) + (2*(a**2)*b - v1**2)*(v2**2) + (a**2)*((b**2)+(c**2))
        f_v2_prime = 4*(a**2)*(v2**3) + 2*(2*(a**2)*b - v1**2)*v2
        v2 -= f_v2 / f_v2_prime
        iterations += 1
    theta2 = math.asin((a*c) / (v1*v2))
    theta2_deg = theta2 * 180 / math.pi
    v2 *= complex(math.cos(theta2), math.sin(theta2))
    i = (v1/a - v2) / (z*(t2**2))
    s1 = (v1/a) * i.conjugate()
    mag_s1 = 100.0 * abs(s1)
    s1_p = 100.0 * s1.real
    s1_q = 100.0 * s1.imag
    s2 = 100 * complex(p, q)
    mag_i1 = 1000 * mag_s1 / (v1*kv1*math.sqrt(3))
    mag_i2 = 1000 * abs(s2) / (abs(v2)*kv2 * math.sqrt(3))
    return [abs(v2), theta2_deg, s1_p, s1_q, abs(i), mag_i1, mag_i2, iterations]

# =====
# ===== MAIN =====
# =====
if __name__ == '__main__':
    KV1 = 138.0
    KV2 = 69.0
    V1 = 0.980
    P = 120.0
    Q = 40.0
    R = 0.00133
    X = 0.039980
    T1 = 0.9875
    T2 = 1.025
    V2,Theta2,S1_P,S1_Q,I,I1,I2,Iterations = get_tap_flow_data(KV1,KV2,V1,T1,T2,R,X,P,Q)
    print '\n|V2| =', V2
    print u'\u03B8'+'\u00B2 = ', Theta2
    print 'P1 = ', S1_P
    print 'Q1 = ', S1_Q
    print '|Ipu| = ', I
    print '|I1| = ', I1
    print '|I2| = ', I2
    if Iterations > 98:
        print 'DID NOT CONVERGE'
```

# Parallel Transformer Flow with Taps



Choose  $V_1$  as reference and analyze side 2

Use Thevenin Equivalent...

$$\overline{V}_{TH} = \frac{|V_1|}{a_1} - \left[ \frac{|V_1|}{a_1} - \frac{|V_1|}{a_2} \right] \left[ \frac{t_{12}^2 \overline{Z}_1}{t_{12}^2 \overline{Z}_1 + t_{22}^2 \overline{Z}_2} \right] = |V_{TH}| \angle \beta$$

$$\overline{Z}_{TH} = t_{12}^2 \overline{Z}_1 \parallel t_{22}^2 \overline{Z}_2 = \frac{t_{12}^2 \overline{Z}_1 t_{22}^2 \overline{Z}_2}{t_{12}^2 \overline{Z}_1 + t_{22}^2 \overline{Z}_2} = R_{TH} + jX_{TH}$$

since these are known...

solve on the side for  $\mathbf{V}_{TH}$  and  $\mathbf{Z}_{TH}$

## Parallel Transformer Flow with Taps (cont.)

$$\vec{I} = \frac{\overrightarrow{V_{TH}} - |V_2|\angle\theta_2}{\overrightarrow{Z_{TH}}} \quad \overrightarrow{S_2} = \vec{I}^* |V_2|\angle\theta_2 \quad \text{where:}$$

$$\vec{I} = \frac{\overrightarrow{S_2}^*}{|V_2|\angle -\theta_2} \quad \overrightarrow{S_2} = P + jQ \text{ (Load)}$$

or:

$$\overrightarrow{S_2} = \overrightarrow{S_{12}} + \overrightarrow{S_{22}}$$

$$\frac{|V_{TH}|\angle\beta - |V_2|\angle\theta_2}{R_{TH} + jX_{TH}} = \frac{P - jQ}{|V_2|\angle -\theta_2}$$

$$|V_{TH}||V_2|\angle(\beta - \theta_2) - |V_2|^2 = (P - jQ)(R_{TH} + jX_{TH})$$

$$|V_{TH}||V_2|\angle(\beta - \theta_2) - |V_2|^2 = \underbrace{(PR_{TH} + QX_{TH})}_E + j\underbrace{(PX_{TH} - QR_{TH})}_F$$

since these are known... make a substitution

$$|V_{TH}||V_2|\angle(\beta - \theta_2) - |V_2|^2 = E + jF$$

$$|V_{TH}||V_2|\cos(\beta - \theta_2) + j|V_{TH}||V_2|\sin(\beta - \theta_2) - |V_2|^2 = E + jF$$

Equating Real and Imaginary terms:

$$|V_{TH}||V_2|\cos(\beta - \theta_2) - |V_2|^2 = E \quad |V_{TH}||V_2|\sin(\beta - \theta_2) = F$$

$$\cos(\beta - \theta_2) = \frac{E + |V_2|^2}{|V_{TH}||V_2|} \quad \sin(\beta - \theta_2) = \frac{F}{|V_{TH}||V_2|}$$

$$\cos^2(\beta - \theta_2) = \frac{|V_2|^4 + 2E|V_2|^2 + E^2}{|V_{TH}|^2|V_2|^2} \quad \sin^2(\beta - \theta_2) = \frac{F^2}{|V_{TH}|^2|V_2|^2}$$

$$\cos^2(\beta - \theta_2) + \sin^2(\beta - \theta_2) = 1$$

## Parallel Transformer Flow with Taps (cont.)

$$\frac{|V_2|^4 + 2E|V_2|^2 + E^2}{|V_{TH}|^2|V_2|^2} + \frac{F^2}{|V_{TH}|^2|V_2|^2} = 1$$

$$|V_2|^4 + 2E|V_2|^2 + E^2 + F^2 = |V_{TH}|^2|V_2|^2$$

$$|V_2|^4 + 2E|V_2|^2 + E^2 + F^2 - |V_{TH}|^2|V_2|^2 = 0$$

$$|V_2|^4 + 2E|V_2|^2 - |V_{TH}|^2|V_2|^2 + E^2 + F^2 = 0$$

$$|V_2|^4 + (2E - |V_{TH}|^2)|V_2|^2 + E^2 + F^2 = 0$$

- 4<sup>th</sup> order equation
- numerical solution required
- will solve with Newton – Raphson Python Script
- Newton – Raphson requires the derivative:

$$4|V_2|^3 + 2(2E - |V_{TH}|^2)[V_2] + E^2 + F^2 = 0$$

After  $|V_2|$  is found...

$$\theta_2 = \beta - \sin^{-1} \left( \frac{F}{|V_{TH}||V_2|} \right)$$

$$\vec{I}_1 = \frac{\frac{|V_1|}{a_1} - V_2 \angle \theta_2}{t_{12}^2 (R_1 + jX_1)}$$

$$\vec{I}_2 = \frac{\frac{|V_1|}{a_2} - V_2 \angle \theta_2}{t_{22}^2 (R_2 + jX_2)}$$

$$S_{12} = \vec{I}_1^* |V_2| \angle \theta_2$$

$$S_{22} = \vec{I}_2^* |V_2| \angle \theta_2$$

$$S_{11} = \frac{|V_1|}{a_1} \vec{I}_1^*$$

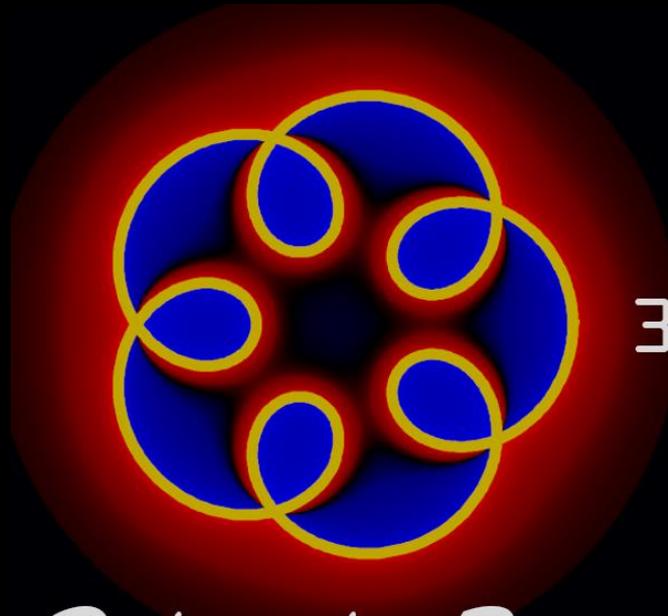
$$S_{21} = \frac{|V_1|}{a_2} \vec{I}_2^*$$

# Newton – Raphson solution

```
import math
import cmath
def get_parallel_tap_flow_data(kv1,kv2,v1,t11,t12,t21,t22,r1,x1,r2,x2,p,q):
    v2 = 1.0 # first guess at v2
    p /= 100.0 # convert to pu
    q /= 100.0 # convert to pu
    a1 = t11 / t12 # calculate net tap ratio
    a2 = t21 / t22 # calculate net tap ratio
    z1 = complex(r1, x1) # define complex xfmr impedance
    z2 = complex(r2, x2) # define complex xfmr impedance
    vth = v1/a1 - (v1/a1 - v1/a2)*((t12**2)*z1)/((t12**2)*z1 + (t22**2)*z2) # calculate vth
    zth = (t12**2)*z1 * (t22**2)*z2 / ((t12**2)*z1 + (t22**2)*z2) # calculate zth
    rth = zth.real # thevenin resistance
    xth = zth.imag # thevenin reactance
    beta = cmath.phase(vth) # vth angle
    e = p*rth + q*xth # substitution variable
    f = p*xth - q*rth # substitution variable
    vth = abs(vth) # magnitude of vth
    iterations = 0 # iteration variable
    tol = 1e-5 # iteration variable
    max_iterations = 99 # iteration variable
    f_v2 = (v2 ** 4) + ((2 * e) - (vth ** 2)) * (v2 ** 2) + (e ** 2) + (f ** 2) # initial v2 solution with guess
    while abs(f_v2) > tol and iterations < max_iterations: # loop while v2 solution != 0
        f_v2 = (v2 ** 4) + ((2 * e) - (vth ** 2)) * (v2 ** 2) + (e ** 2) + (f ** 2) # v2 solution (this iteration)
        f_v2_prime = 4 * (v2 ** 3) + 2 * ((2 * e) - (vth ** 2)) * v2 # derivative of v2 solution
        v2 -= f_v2 / f_v2_prime # adjust next iteration v2
        iterations += 1 # increment counter
    theta2 = beta - math.asin(f / (vth * v2)) # calculate bus2 angle (radians)
    theta2_deg = theta2 * 180 / math.pi # calculate bus2 angle (degrees)
    v2 *= complex(math.cos(theta2), math.sin(theta2)) # define complex v2
    i1 = (v1/a1 - v2) / ((t12**2)*z1) # calculate pu complex current1
    i2 = (v1/a2 - v2) / ((t22**2)*z2) # calculate pu complex current2
    s11 = (v1/a1) * i1.conjugate() # calculate sending complex power1
    s12 = v2 * i1.conjugate() # calculate receiving complex power2
    s21 = (v1/a2) * i2.conjugate() # calculate sending complex power1
    s22 = v2 * i2.conjugate() # calculate receiving complex power2
    s11_p = 100.0 * s11.real # sending real power1 (MW)
    s11_q = 100.0 * s11.imag # sending reactive power1 (MVAR)
    s12_p = 100.0 * s12.real # receiving real power1 (MW)
    s12_q = 100.0 * s12.imag # receiving reactive power1 (MVAR)
    s21_p = 100.0 * s21.real # sending real power2 (MW)
    s21_q = 100.0 * s21.imag # sending reactive power2 (MVAR)
    s22_p = 100.0 * s22.real # receiving real power2 (MW)
    s22_q = 100.0 * s22.imag # receiving reactive power2 (MVAR)
    mag_s11 = 100.0 * abs(s11) # sending apparent power1 (MW)
    mag_s12 = 100.0 * abs(s12) # receiving apparent power1 (MW)
    mag_s21 = 100.0 * abs(s21) # sending apparent power2 (MW)
    mag_s22 = 100.0 * abs(s22) # receiving apparent power2 (MW)
    v2 = abs(v2) # use magnitude of v2 for remaining
    mag_i11 = 1000 * mag_s11 / (v1*kv1*math.sqrt(3)) # calculate magnitude of current11
    mag_i12 = 1000 * mag_s12 / (v2*kv2 * math.sqrt(3)) # calculate magnitude of current12
    mag_i21 = 1000 * mag_s21 / (v1*kv1*math.sqrt(3)) # calculate magnitude of current21
    mag_i22 = 1000 * mag_s22 / (v2*kv2 * math.sqrt(3)) # calculate magnitude of current22
    return [v2, theta2_deg, s11_p, s11_q, s12_p, s12_q, s21_p, s21_q, s22_p, s22_q, mag_i11, mag_i12, mag_i21, mag_i22, iterations]
```

## Newton – Raphson solution (cont.)

```
# =====  
# ===== MAIN =====  
# =====  
if __name__ == '__main__':  
    kv1 = 138.0 # <--- Input Bus1 Base kV  
    kv2 = 69.0 # <--- Input Bus2 Base kV  
    v1 = 0.980 # <--- Input Bus1 Voltage (pu)  
    r1 = 0.00266 # <--- Input Xfmr1 Resistance (pu)  
    x1 = 0.07996 # <--- Input Xfmr1 Reactance (pu)  
    r2 = 0.00266 # <--- Input Xfmr2 Resistance (pu)  
    x2 = 0.07996 # <--- Input Xfmr2 Reactance (pu)  
    t11 = 0.9875 # <--- Input Xfmr1 Tap1 (pu)  
    t12 = 1.025 # <--- Input Xfmr1 Tap2 (pu)  
    t21 = 0.9875 # <--- Input Xfmr2 Tap1 (pu)  
    t22 = 1.025 # <--- Input Xfmr2 Tap2 (pu)  
    p = 120.0 # <--- Input Load Real Power (MW)  
    q = 40.0 # <--- Input Load Reactive Power (MVAR)  
    data = get_parallel_tap_flow_data(kv1, kv2, v1, t11, t12, t21, t22, r1, x1, r2, x2, p, q)  
    print ('----- XFMR 1 -----')  
    print ('V1 ='), (v1)  
    print ('TAP1 ='), (t11)  
    print ('P1 ='), (data[2])  
    print ('Q1 ='), (data[3])  
    print ('I1 ='), (data[10])  
    print ('\nTAP2 ='), (t12)  
    print ('P2 ='), (data[4])  
    print ('Q2 ='), (data[5])  
    print ('I2 ='), (data[11])  
    print ('V2 ='), (data[0])  
    print ('THETA2 ='), (data[1])  
    print ('\n----- XFMR 2 -----')  
    print ('V1 ='), (v1)  
    print ('TAP1 ='), (t21)  
    print ('P1 ='), (data[6])  
    print ('Q1 ='), (data[7])  
    print ('I1 ='), (data[12])  
    print ('\nTAP2 ='), (t22)  
    print ('P2 ='), (data[8])  
    print ('Q2 ='), (data[9])  
    print ('I2 ='), (data[13])  
    print ('V2 ='), (data[0])  
    print ('THETA2 ='), (data[1])  
    if data[14] > 98:  
        print ('DID NOT CONVERGE')
```



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Questions or Comments ...

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