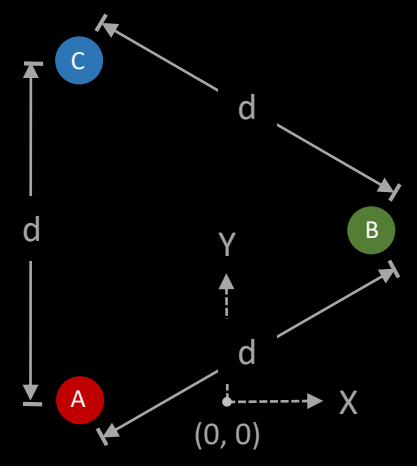
Calculating 3Φ T-Line Magnetic Fields
(Equilateral Configuration)

consider a 3Φ circuit with infinity long straight conductors

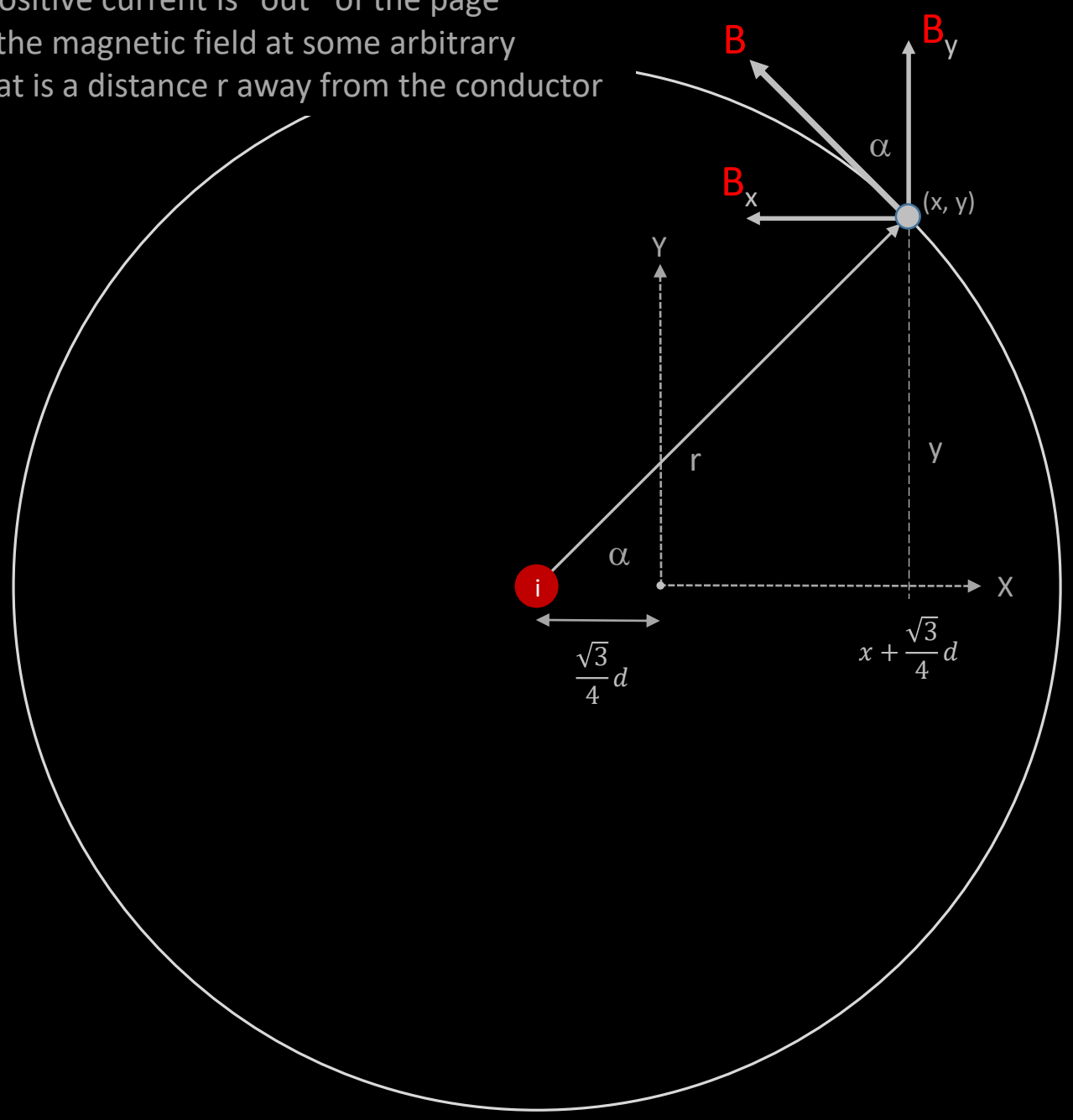


choose x origin of coordinate system at center of the single pole
choose y origin of coordinate system at center of phase A
Let the spacing between the conductors = d

first consider the A phase conductor
then continue with B and C phase using superposition

Calculating 3Φ T-Line Magnetic Fields

Let the instantaneous current flowing in the conductor = i
where positive current is "out" of the page
consider the magnetic field at some arbitrary
point = (x, y) that is a distance r away from the conductor



The magnitude of the magnetic field
is given by Ampere's law

$$B = \frac{\mu_0 i}{2\pi r}$$

Permeability of Free Space (air)

$$\mu_0 = 4\pi \times 10^{-7}\frac{H}{m}$$

$$\mu_0 = 3.83023 \times 10^{-7}\frac{H}{ft}$$

calculate r

$$r = \sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}$$

$$B = \frac{\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$

Note: when the current is negative or "into" the page, the B field will point in the opposite direction

Calculating 3Φ T-Line Magnetic Fields

Next we want to know the horizontal and vertical components of B

$$B = \frac{\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$

$$B_x = \frac{-\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}\sin\alpha$$

$$B_y = \frac{\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}\cos\alpha$$

where:

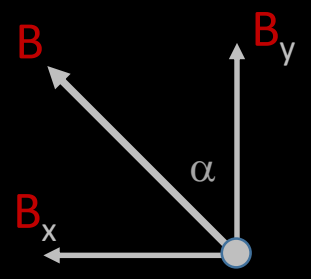
$$\sin\alpha = \frac{y}{r} = \frac{y}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$

$$\cos\alpha = \frac{x}{r} = \frac{x + \frac{\sqrt{3}}{4}d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$

$\therefore$

$$B_x = \frac{-\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}\frac{y}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$

$$B_y = \frac{\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}\frac{x + \frac{\sqrt{3}}{4}d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$



finally:

$$B_x = \frac{-\mu_0 i y}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2\right]}$$

$$B_y = \frac{\mu_0 i\left[x + \frac{\sqrt{3}}{4}d\right]}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2\right]}$$

Since this derivation was for A phase…
modify the notation slightly:

$$\boxed{\begin{aligned} B_{Ax} &= \frac{-\mu_0 i_A y}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2\right]} \\ B_{Ay} &= \frac{\mu_0 i_A\left[x + \frac{\sqrt{3}}{4}d\right]}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2\right]} \end{aligned}}$$

Calculating 3Φ T-Line Magnetic Fields

Now let's repeat for B phase

$$B = \frac{\mu_0 i}{2\pi\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}$$

$$B_x = \frac{-\mu_0 i}{2\pi\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}\sin\alpha$$

$$B_y = \frac{\mu_0 i}{2\pi\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}\cos\alpha$$

where:

$$\sin\alpha = \frac{y}{r} = \frac{y - \frac{d}{2}}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}$$

$$\cos\alpha = \frac{x}{r} = \frac{x - \frac{\sqrt{3}}{4}d}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}$$

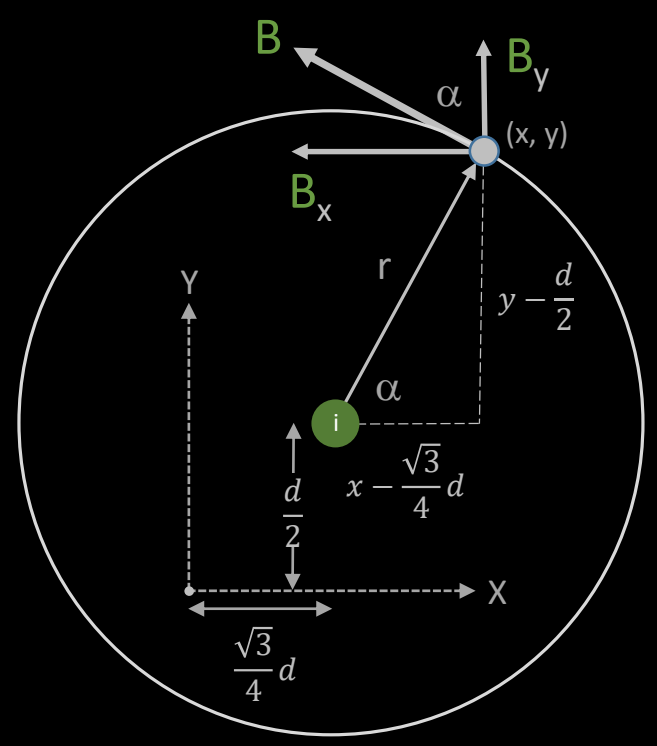
$\therefore$

$$B_x = \frac{-\mu_0 i}{2\pi\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}\frac{y - \frac{d}{2}}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}$$

$$B_y = \frac{\mu_0 i}{2\pi\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}\frac{x - \frac{\sqrt{3}}{4}d}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}$$

$$B = \frac{\mu_0 i}{2\pi r}$$

$$r = \sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}$$



$$B_{Bx} = \frac{-\mu_0 i_B\left[y - \frac{d}{2}\right]}{2\pi\left[\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2\right]}$$

$$B_{By} = \frac{\mu_0 i_B\left[x - \frac{\sqrt{3}}{4}d\right]}{2\pi\left[\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2\right]}$$

Calculating 3Φ T-Line Magnetic Fields

Finally… repeat for C phase

$$B = \frac{\mu_0 i}{2\pi r}$$

$$B = \frac{\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2}}$$

$$B_x = \frac{-\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2}}\sin\alpha$$

$$B_y = \frac{\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2}}\cos\alpha$$

where:

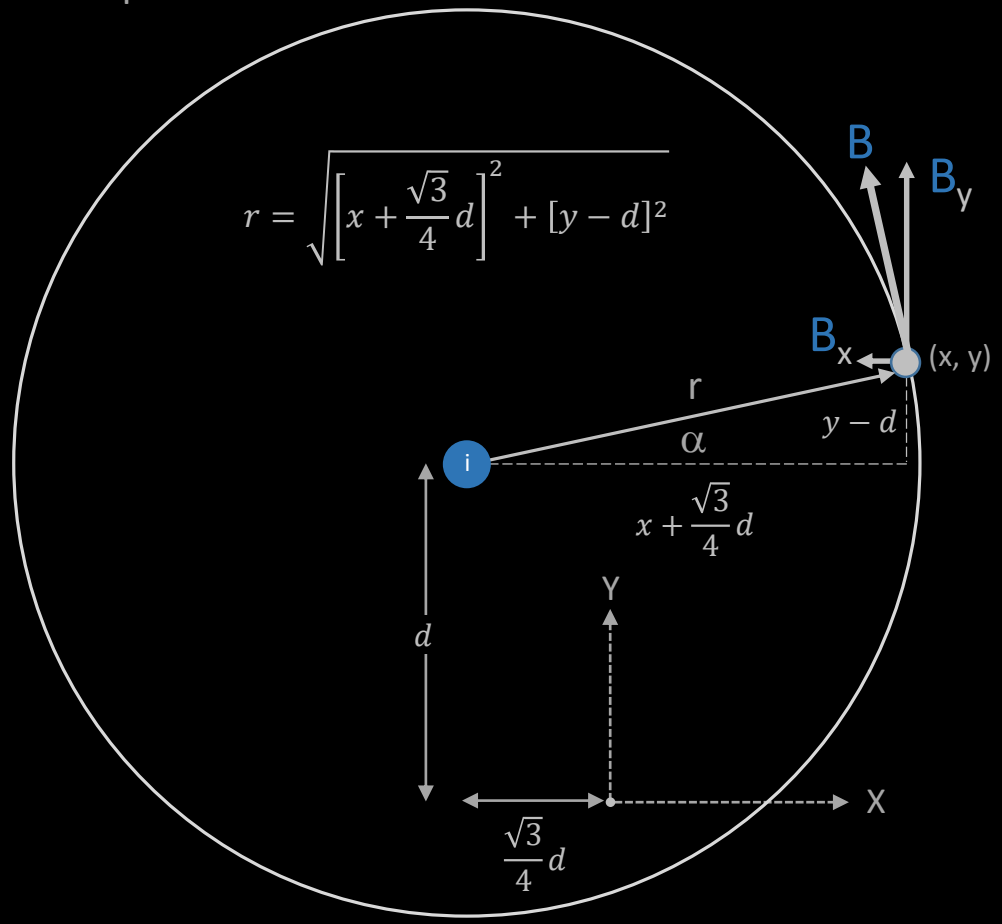
$$\sin\alpha = \frac{y}{r} = \frac{y-d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2}}$$

$$\cos\alpha = \frac{x}{r} = \frac{x + \frac{\sqrt{3}}{4}d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2}}$$

$\therefore$

$$B_x = \frac{-\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2}}\frac{y-d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2}}$$

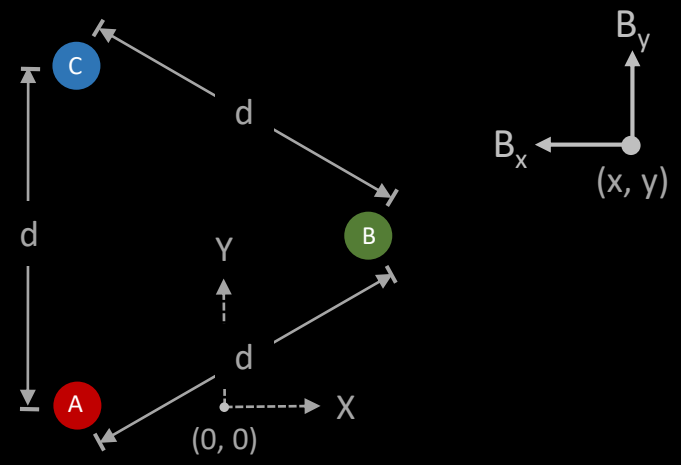
$$B_y = \frac{\mu_0 i}{2\pi\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2}}\frac{x + \frac{\sqrt{3}}{4}d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2}}$$



$$\boxed{\begin{aligned} B_{Cx} &= \frac{-\mu_0 i_C [y-d]}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2\right]} \\ B_{Cy} &= \frac{\mu_0 i_C \left[x + \frac{\sqrt{3}}{4}d\right]}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y-d]^2\right]} \end{aligned}}$$

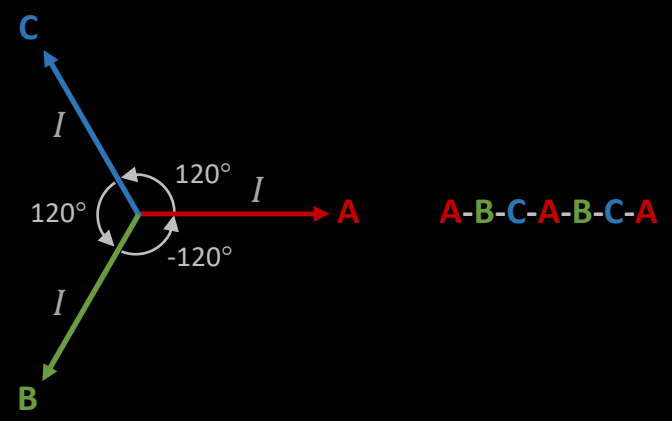
Calculating 3Φ T-Line Magnetic Fields

so far we have the B field at any arbitrary point at single instant in time…



Substitute the Time Dependent AC Current for each Phase

for a balanced 3Φ system in positive sequence…
all three phases have the same RMS current amplitude = I
rotating counter clockwise
and are out of phase by 120°



$$i_A = I\cos(\omega t)$$
$$i_B = I\cos(\omega t - 120°)$$
$$i_C = I\cos(\omega t + 120°)$$
$$\omega = 2\pi f = \theta$$

$$i_A = I\cos(\theta)$$
$$i_B = I\cos(\theta - 120°)$$
$$i_C = I\cos(\theta + 120°)$$
$$\omega = 2\pi f$$

Calculating 3Φ T-Line Magnetic Fields

substitute alternating phase currents and simplify

$$i_A = I\cos(\theta) \qquad i_B = I\cos(\theta - 120°) \qquad i_C = I\cos(\theta + 120°)$$

Horizontal Components

$$B_{Ax} = \frac{-\mu_0 i_A y}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2\right]} = \frac{-\mu_0 y I\cos(\theta)}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2\right]} = -\left[\frac{\mu_0 I}{2\pi}\right]\frac{y\cos(\theta)}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}$$

$$B_{Bx} = \frac{-\mu_0 i_B\left[y - \frac{d}{2}\right]}{2\pi\left[\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2\right]} = \frac{-\mu_0\left[y - \frac{d}{2}\right] I\cos(\theta - 120°)}{2\pi\left[\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2\right]} = -\left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[y - \frac{d}{2}\right]\cos(\theta - 120°)}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}$$

$$B_{Cx} = \frac{-\mu_0 i_C[y - d]}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2\right]} = \frac{-\mu_0[y - d] I\cos(\theta + 120°)}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2\right]} = -\left[\frac{\mu_0 I}{2\pi}\right]\frac{[y - d]\cos(\theta + 120°)}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}$$

Vertical Components

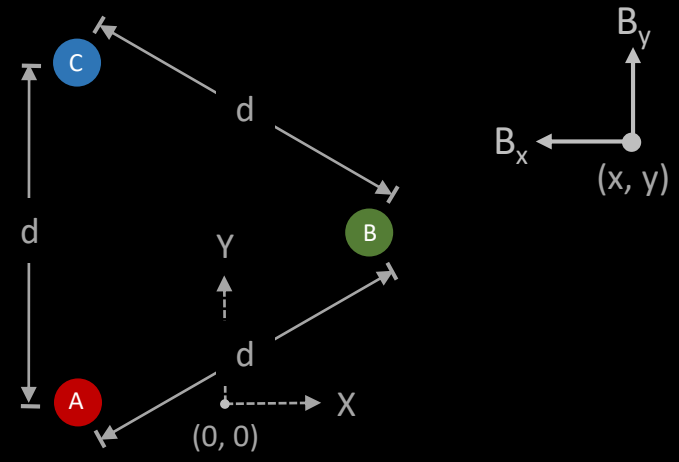
$$B_{Ay} = \frac{\mu_0 i_A\left[x + \frac{\sqrt{3}}{4}d\right]}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2\right]} = \frac{\mu_0\left[x + \frac{\sqrt{3}}{4}d\right] I\cos(\theta)}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2\right]} = \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[x + \frac{\sqrt{3}}{4}d\right]\cos(\theta)}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}$$

$$B_{By} = \frac{\mu_0 i_B\left[x - \frac{\sqrt{3}}{4}d\right]}{2\pi\left[\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2\right]} = \frac{\mu_0\left[x - \frac{\sqrt{3}}{4}d\right] I\cos(\theta - 120°)}{2\pi\left[\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2\right]} = \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[x - \frac{\sqrt{3}}{4}d\right]\cos(\theta - 120°)}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}$$

$$B_{Cy} = \frac{\mu_0 i_C\left[x + \frac{\sqrt{3}}{4}d\right]}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2\right]} = \frac{\mu_0\left[x + \frac{\sqrt{3}}{4}d\right] I\cos(\theta + 120°)}{2\pi\left[\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2\right]} = \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[x + \frac{\sqrt{3}}{4}d\right]\cos(\theta + 120°)}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}$$

Calculating 3Φ T-Line Magnetic Fields

Now add A B C phase contributions together to get total B field at any point (x, y) and any time



$$B_{Ax} = -\left[\frac{\mu_0 I}{2\pi}\right]\frac{y\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2}$$

$$B_{Ay} = \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2}$$

$$B_{Bx} = -\left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[y-\frac{d}{2}\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2}$$

$$B_{By} = \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[x-\frac{\sqrt{3}}{4}d\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2}$$

$$B_{Cx} = -\left[\frac{\mu_0 I}{2\pi}\right]\frac{[y-d]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}$$

$$B_{Cy} = \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}$$

$$B_x = B_{Ax} + B_{Bx} + B_{Cx}$$

$$\boxed{B_x = -\left[\frac{\mu_0 I}{2\pi}\right]\frac{y\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2} - \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[y-\frac{d}{2}\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2} - \left[\frac{\mu_0 I}{2\pi}\right]\frac{[y-d]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}}$$

$$B_y = B_{Ay} + B_{By} + B_{Cy}$$

$$\boxed{B_y = \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2} + \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[x-\frac{\sqrt{3}}{4}d\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2} + \left[\frac{\mu_0 I}{2\pi}\right]\frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}}$$

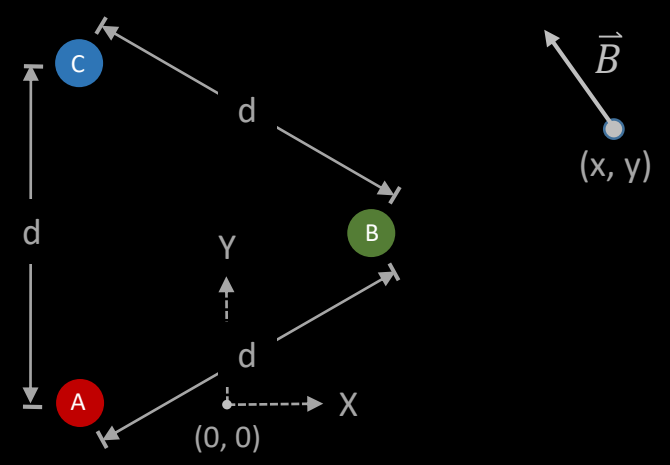
Calculating 3Φ T-Line Magnetic Fields

Simplify

$$B_x = -\frac{\mu_0 I}{2\pi}\left[\frac{y\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2}+\frac{\left[y-\frac{d}{2}\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2}+\frac{[y-d]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}\right]$$

$$B_y = \frac{\mu_0 I}{2\pi}\left[\frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2}+\frac{\left[x-\frac{\sqrt{3}}{4}d\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2}+\frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}\right]$$

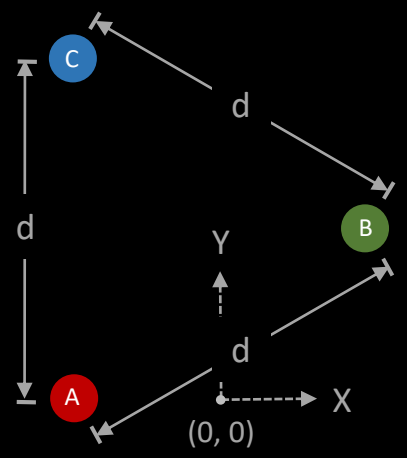
$$\vec{B} = B_x\hat{x} + B_y\hat{y}$$



You could plot the B field vectors for any number of points in space for any instant in time.
The length of the vector would indicate the magnetic field magnitude.
The direction of the vector would indicate if the field is positive or negative.
This method of illustrating fields is not very exciting…. Let's consider another way

Animating 3Φ T-Line Magnetic Fields

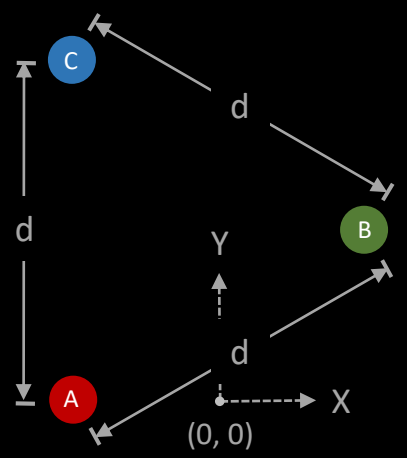
For this exercise we would like to animate the magnitude of the B field = $|B|$

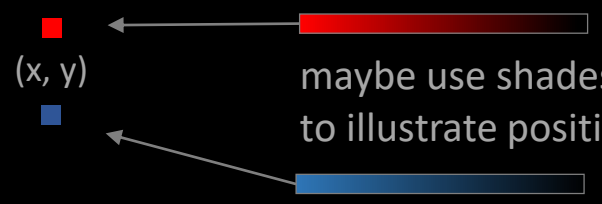


maybe use shades of red to illustrate B field intensity

$$|B| = \sqrt{B_x^2 + B_y^2}$$

The above method would be a good start but it would not give an indication of the polarity or direction of the field. We want to illustrate positive field intensity with one color and negative field intensity with another color





maybe use shades of red to illustrate positive B field intensity

and shades of blue to illustrate negative B field intensity

Animating 3Φ T-Line Magnetic Fields

It would seem that we have all the information we need to determine the polarity, But …
Ampere's law assumes you know the current direction, then … use the right-hand rule to get the B field direction.
That works great if you have only one conductor.
We need to:

1) find the contribution (with sign) from each phase
2) use superposition to add up the contributions
3) determine if the net contribution is positive or negative

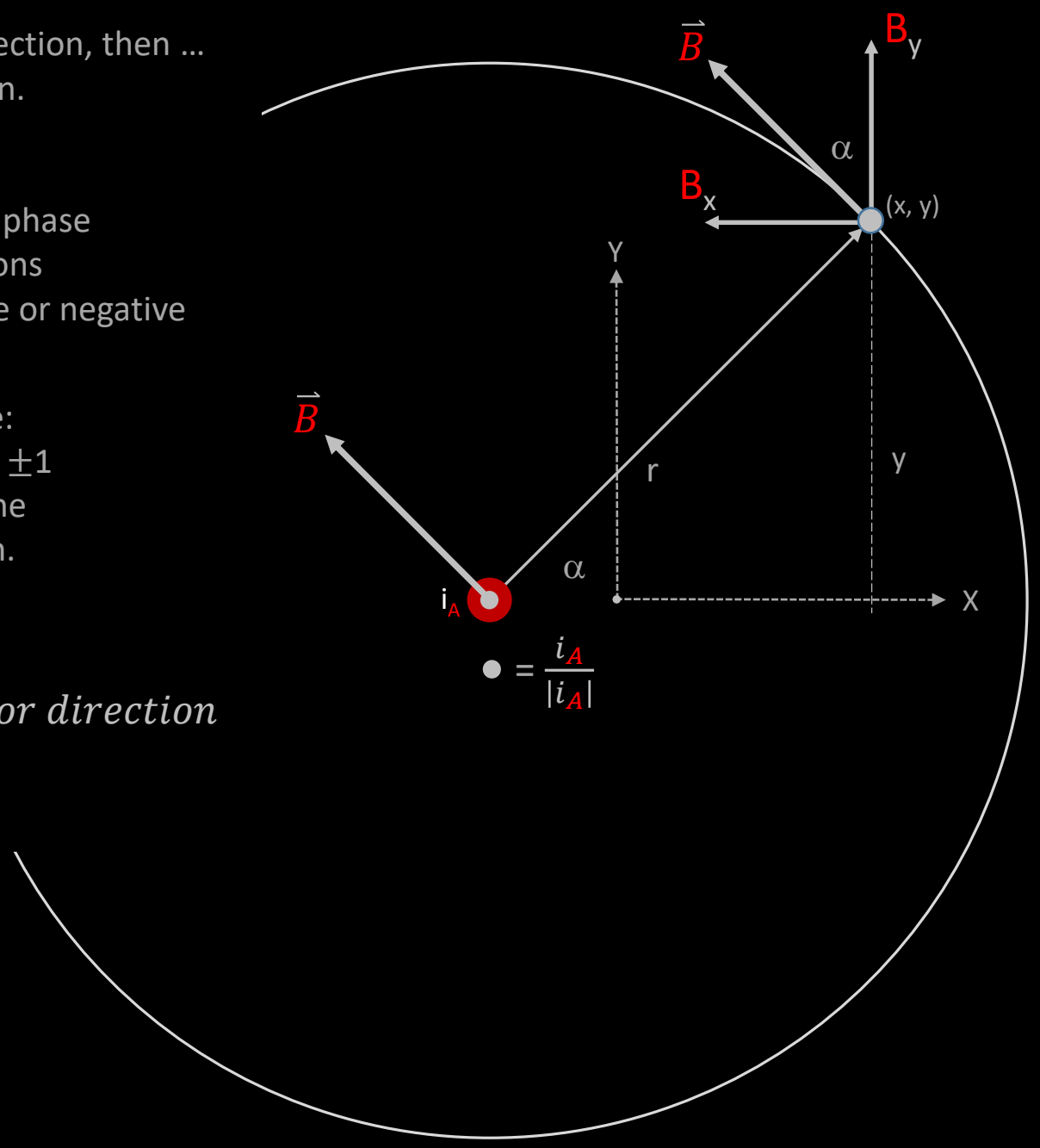
Returning to the previous derivation…
and using the A phase conductor as and example:
We could use product of the magnitude of B and ± 1 (depending on current direction) to determine the contribution of the B field to polarity or direction.
we can get ± 1 with $\frac{i_B}{|i_B|}$

$$let\ \delta_B = contribution\ of\ \vec{B}\ to\ polarity\ or\ direction$$

$$\therefore\ \delta_B = |\vec{B}|\frac{i_B}{|i_B|}$$

Since this calculation was for A phase…
modify the notation slightly:

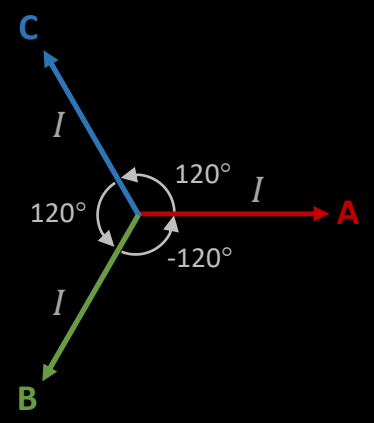
$$\delta_B = |\vec{B}_B|\frac{i_B}{|i_B|}$$



Animating 3Φ T-Line Magnetic Fields

continue with the sign or polarity of A and C phase

$$i_A = I\cos(\theta)$$
$$i_B = I\cos(\theta - 120°)$$
$$i_C = I\cos(\theta + 120°)$$



substituting ABC alternating phase currents…

$$\delta_A = |\vec{B}_A|\frac{i_A}{|i_A|} = |\vec{B}_A|\frac{I\cos(\theta)}{|I\cos(\theta)|} = |\vec{B}_A|\frac{\cos(\theta)}{|\cos(\theta)|}$$

$$\delta_B = |\vec{B}_B|\frac{i_B}{|i_B|} = |\vec{B}_B|\frac{I\cos(\theta - 120°)}{|I\cos(\theta - 120°)|} = |\vec{B}_B|\frac{\cos(\theta - 120°)}{|\cos(\theta - 120°)|}$$

$$\delta_C = |\vec{B}_C|\frac{i_C}{|i_C|} = |\vec{B}_C|\frac{I\cos(\theta + 120°)}{|I\cos(\theta + 120°)|} = |\vec{B}_C|\frac{\cos(\theta + 120°)}{|\cos(\theta + 120°)|}$$

$$\delta_A = |\vec{B}_A|\frac{\cos(\theta)}{|\cos(\theta)|} \qquad \delta_B = |\vec{B}_B|\frac{\cos(\theta - 120°)}{|\cos(\theta - 120°)|} \qquad \delta_C = |\vec{B}_C|\frac{\cos(\theta + 120°)}{|\cos(\theta + 120°)|}$$

Animating 3Φ T-Line Magnetic Fields

get the total contribution to the B field

$$\delta_A = |\vec{B}_A| \frac{\cos(\theta)}{|\cos(\theta)|} \qquad \delta_B = |\vec{B}_B| \frac{\cos(\theta - 120°)}{|\cos(\theta - 120°)|} \qquad \delta_C = |\vec{B}_C| \frac{\cos(\theta + 120°)}{|\cos(\theta + 120°)|}$$

total contribution to the resulting $\vec{\mathrm{B}}$ field $= \delta_A + \delta_B + \delta_C$

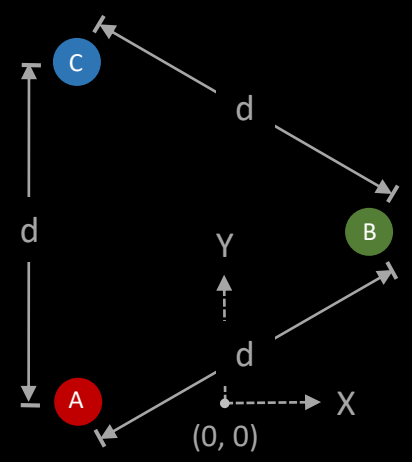
but we are only interested in the sign (± 1) of the total contribution

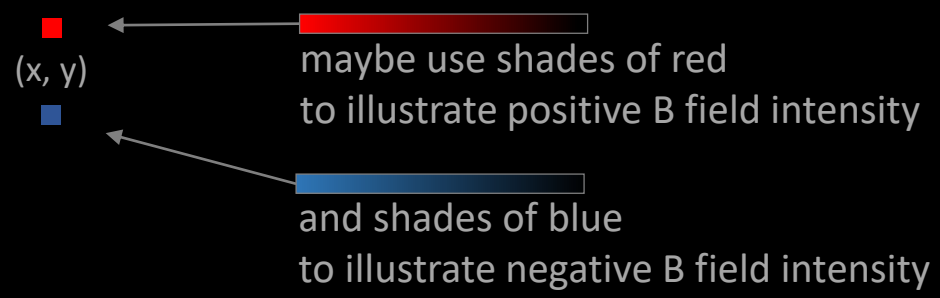
the sign (± 1) of the total contributionto the resulting $\vec{\mathrm{B}}$ field $= \dfrac{\delta_A + \delta_B + \delta_C}{|\delta_A + \delta_B + \delta_C|}$

now we can add the polarity or direction to the magnitude of the B field

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2} \qquad \vec{B}_{polarity} = \frac{\delta_A + \delta_B + \delta_C}{|\delta_A + \delta_B + \delta_C|}$$

we can now plot $B(x, y, d, \theta) = \dfrac{\delta_A + \delta_B + \delta_C}{|\delta_A + \delta_B + \delta_C|} \sqrt{B_x^2 + B_y^2}$





$$B = \frac{\delta_A + \delta_B + \delta_C}{|\delta_A + \delta_B + \delta_C|} \sqrt{B_x^2 + B_y^2}$$

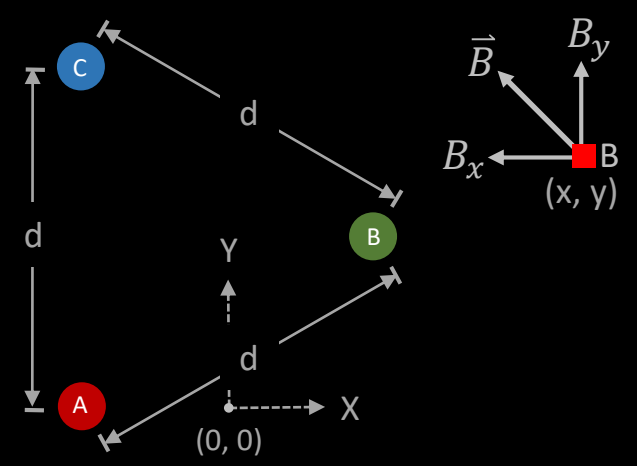
Summary So Far

$$B_x = -\underbrace{\frac{\mu_0 I}{2\pi}}_{\text{normalize and drop}}\left[\underbrace{\frac{y\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2}}_{B_{Ax}}+\underbrace{\frac{\left[y-\frac{d}{2}\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2}}_{B_{Bx}}+\underbrace{\frac{[y-d]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}}_{B_{Cx}}\right]$$

$$B_y = \frac{\mu_0 I}{2\pi}\left[\underbrace{\frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2}}_{B_{Ay}}+\underbrace{\frac{\left[x-\frac{\sqrt{3}}{4}d\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2}}_{B_{By}}+\underbrace{\frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}}_{B_{Cy}}\right]$$

$$|\vec{B}_A| = \sqrt{B_{Ax}^2+B_{Ay}^2} \qquad |\vec{B}_B| = \sqrt{B_{Bx}^2+B_{By}^2} \qquad |\vec{B}_C| = \sqrt{B_{Cx}^2+B_{Cy}^2}$$

$$\delta_A = |\vec{B}_A|\frac{\cos(\theta)}{|\cos(\theta)|} \qquad \delta_B = |\vec{B}_B|\frac{\cos(\theta-120°)}{|\cos(\theta-120°)|} \qquad \delta_C = |\vec{B}_C|\frac{\cos(\theta+120°)}{|\cos(\theta+120°)|}$$



$$\boxed{B = \frac{\delta_A+\delta_B+\delta_C}{|\delta_A+\delta_B+\delta_C|}\frac{\sqrt{B_x^2+B_y^2}}{B_{Base}}}$$

now lets find the “Base” to normalize on

Animating 3Φ T-Line Magnetic Fields

find the maximum B field for normalizing

$$B_x = -\left[\frac{y\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2} + \frac{\left[y-\frac{d}{2}\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2} + \frac{[y-d]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}\right]$$

$$B_x = -\left[\frac{s\cos(0)}{\left[-\frac{\sqrt{3}}{4}d+\frac{\sqrt{3}}{4}d\right]^2+s^2} + \frac{\left[s-\frac{d}{2}\right]\cos(-120°)}{\left[-\frac{\sqrt{3}}{4}d-\frac{\sqrt{3}}{4}d\right]^2+\left[s-\frac{d}{2}\right]^2} + \frac{[s-d]\cos(120°)}{\left[-\frac{\sqrt{3}}{4}d+\frac{\sqrt{3}}{4}d\right]^2+[s-d]^2}\right]$$

$$B_x = -\left[\frac{s}{s^2} + \frac{-0.5\left[s-\frac{d}{2}\right]}{\left[\frac{\sqrt{3}}{2}d\right]^2+\left[s-\frac{d}{2}\right]^2} + \frac{-0.5[s-d]}{[s-d]^2}\right] = -\left[\frac{1}{s} + \frac{\frac{d}{2}-s}{2\left[\frac{\sqrt{3}}{2}d\right]^2+2\left[\frac{d}{2}-s\right]^2} + \frac{1}{2[d-s]}\right]$$

$$B_x = -\left[\frac{1}{s} + \frac{d-2s}{4\left[\frac{\sqrt{3}}{2}d\right]^2+4\left[\frac{d}{2}-s\right]^2} + \frac{1}{2[d-s]}\right] = -\left[\frac{1}{s} + \frac{d-2s}{3d^2+[d-2s]^2} + \frac{1}{2[d-s]}\right]$$

$$B_y = \left[\frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+y^2} + \frac{\left[x-\frac{\sqrt{3}}{4}d\right]\cos(\theta-120°)}{\left[x-\frac{\sqrt{3}}{4}d\right]^2+\left[y-\frac{d}{2}\right]^2} + \frac{\left[x+\frac{\sqrt{3}}{4}d\right]\cos(\theta+120°)}{\left[x+\frac{\sqrt{3}}{4}d\right]^2+[y-d]^2}\right]$$

$$B_y = \left[\frac{\left[-\frac{\sqrt{3}}{4}d+\frac{\sqrt{3}}{4}d\right]\cos(0)}{\left[-\frac{\sqrt{3}}{4}d+\frac{\sqrt{3}}{4}d\right]^2+s^2} + \frac{\left[-\frac{\sqrt{3}}{4}d-\frac{\sqrt{3}}{4}d\right]\cos(-120°)}{\left[-\frac{\sqrt{3}}{4}d-\frac{\sqrt{3}}{4}d\right]^2+\left[s-\frac{d}{2}\right]^2} + \frac{\left[-\frac{\sqrt{3}}{4}d+\frac{\sqrt{3}}{4}d\right]\cos(120°)}{\left[-\frac{\sqrt{3}}{4}d+\frac{\sqrt{3}}{4}d\right]^2+[s-d]^2}\right]$$

$$B_y = \left[\frac{0\cos(0)}{0^2+s^2} + \frac{\frac{\sqrt{3}}{4}d}{\left[\frac{\sqrt{3}}{2}d\right]^2+\left[s-\frac{d}{2}\right]^2} + \frac{0\cos(120°)}{0^2+[s-d]^2}\right]$$

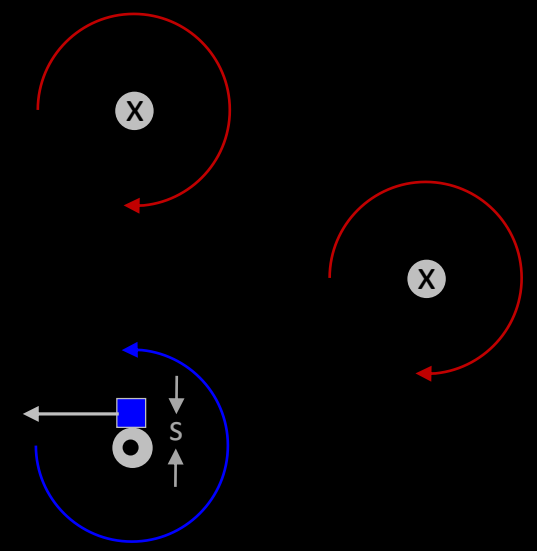
$$B_y = \frac{\sqrt{3}d}{3d^2+[d-2s]^2}$$

consider A phase:
assume theta = 0° (max A phase current)

$x = -\frac{\sqrt{3}}{4}d$ at the center of the A phase conductor

take a very small step in the y direction
(on the order of conductor radius, only inches!)
call this very small step = s

the coordinate of interest is x = $-\frac{\sqrt{3}}{4}d$, y = s



$$B_x^2 = \left[\frac{1}{s} + \frac{d-2s}{3d^2+[d-2s]^2} + \frac{1}{2[d-s]}\right]^2$$

$$B_y^2 = \frac{3d^2}{[3d^2+[d-2s]^2]^2}$$

$$\boxed{B_{Base} = |B|_{max} = \sqrt{B_x^2+B_y^2}}$$

"Base" magnetic field for normalizing
(numeric solution)

pseudo code for animating T-Line Magnetic Fields

```
d = 20.0                                          # define conductor spacing
Bbase = f(d,s)                                    # calculate the max field (for normalizing)

for angle in range(0,361)                         # loop current angles (phase A is reference)
    for x in range(-40,40,0.2)                    # loop x coordinates from -2*d to 2*d in steps of d/100
        for y in range(-40,60,0.2)                # loop y coordinates from -2*d to 3*d in steps of d/100
            angle_a = angle                       # get phase A angle (phase A is reference)
            angle_b = angle_a-120                 # get phase B angle
            angle_c = angle_a+120                 # get phase C angle
            Bax = f(x,y,d,angle_a)                # calculate A phase field in x direction
            Bbx = f(x,y,d,angle_b)                # calculate B phase field in x direction
            Bcx = f(x,y,d,angle_c)                # calculate C phase field in x direction
            Bay = f(x,y,d,angle)                  # calculate A phase field in y direction
            Bby = f(x,y,d,angle_b)                # calculate B phase field in y direction
            Bcy = f(x,y,d,angle_c)                # calculate C phase field in y direction
            |Ba| = pow(Bax^2+Bay^2,0.5)           # calculate magnitude A phase field
            |Bb| = pow(Bbx^2+Bby^2,0.5)           # calculate magnitude B phase field
            |Bc| = pow(Bcx^2+Bcy^2,0.5)           # calculate magnitude C phase field
            da = |Ba|*cos(angle_a)/abs(cos(angle_a))  # calculate A phase contribution
            db = |Bb|*cos(angle_b)/abs(cos(angle_b))  # calculate B phase contribution
            dc = |Bc|*cos(angle_c)/abs(cos(angle_c))  # calculate C phase contribution
            polarity = (da+db+dc)/abs(da+db+dc)   # get the sign of the magnetic field
            Bx = Bax+Bbx+Bcx                      # get net field in x direction
            By = Bay+Bby+Bcy                      # get net field in y direction
            |B| = pow(Bx^2+By^2,0.5)              # get magnitude of net field
            B = polarity*|B|/Bbase                # report normalized field magnitude, with sign

            # this nested loop will give you the field at one (x,y) coordinate at one current angle
            # we recommend looking into matplotlib and seaborn for generating xy grid and using heatmaps
```

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Questions or Comments …

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