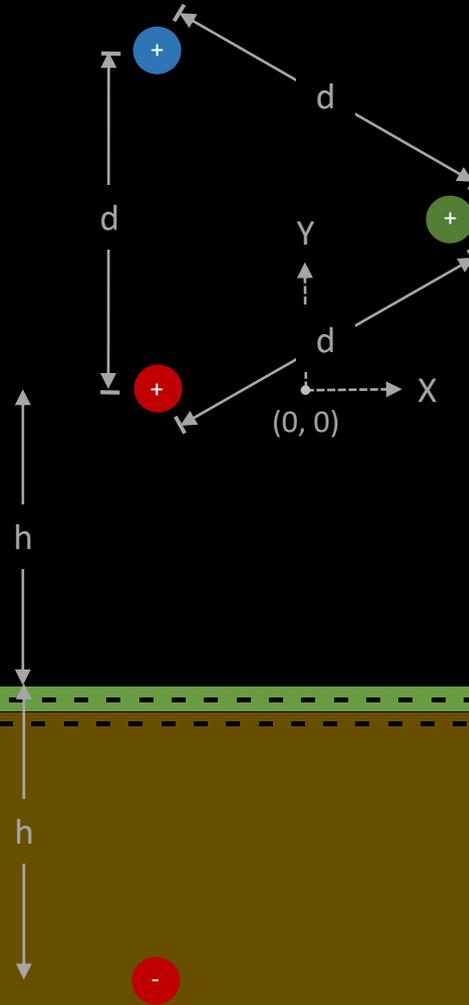


# Calculating 3 $\Phi$ T-Line Electric Fields (Equilateral Configuration)

consider a 3 $\Phi$  circuit with infinity long straight conductors and some height above ground

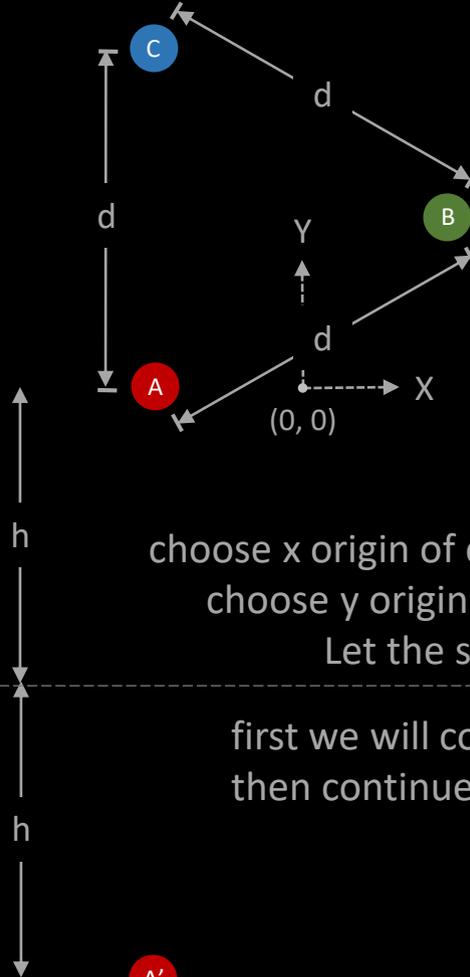
For this problem, the effects of earth need to be considered. Since the earth is a good conductor of electric current... It will be modeled as a conducting plane at the surface.



The electric charge on the lines will create electric fields. The electric fields will induce an opposite charge near the earth's surface.

The concept of "image conductors" are used to calculate the net electric field around the overhead transmission lines.

# Calculating 3 $\Phi$ T-Line Electric Fields (Equilateral Configuration)



choose x origin of coordinate system at center of the single pole  
choose y origin of coordinate system at center of phase A  
Let the spacing between the conductors = d

first we will consider the center A phase conductor  
then continue with B and C phase using superposition

A'

B'

C'

# Calculating 3Φ T-Line Electric Fields

Let the instantaneous current flowing in the conductor =  $i$  where positive current is “out” of the page consider the electric field at some arbitrary point =  $(x, y)$  that is a distance  $r$  away from the **A** conductor

permeability of free space

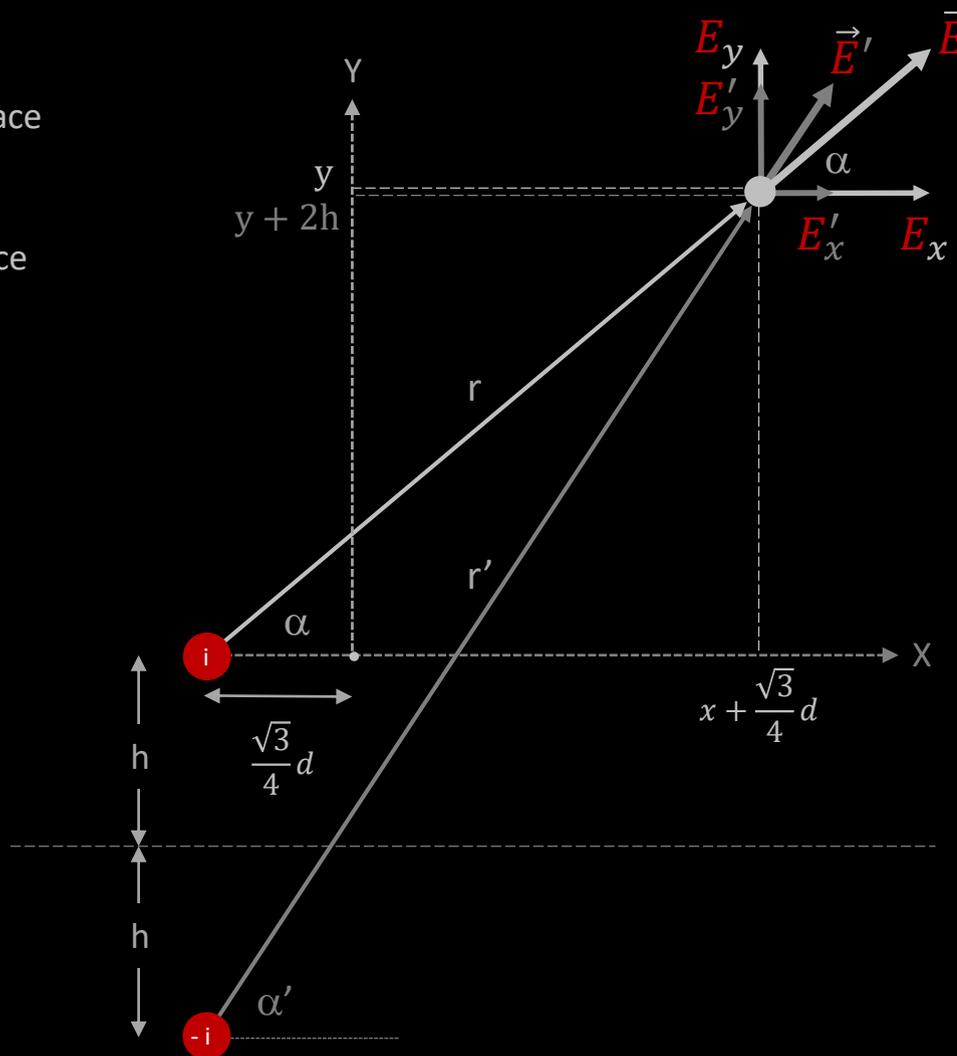
$$\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}$$

permittivity of free space

$$\epsilon_0 = 8.854 \times 10^{-12} \frac{F}{m}$$

line charge density

$$\rho = i\sqrt{\mu_0\epsilon_0}$$



The magnitude of the electric field is given as

$$\vec{E} = \sqrt{\frac{\mu_0}{\epsilon_0} \frac{i}{2\pi r}} = \frac{120\pi i}{2\pi r} = \frac{60i}{r} \hat{r}$$

where  $i$  is the instantaneous RMS current

calculate  $r$

$$r = \sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}$$

calculate  $r'$

$$r' = \sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2}$$

$$\vec{E} = \frac{60i_A}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$

$$\vec{E}' = \frac{-60i_A}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2}}$$

Note: when the current is negative or “into” the page, the E field will point in the opposite direction

## Calculating 3Φ T-Line Electric Fields

next we want to know the horizontal and vertical components of E and E' for A phase

$$\vec{E} = \frac{60i_A}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$

$$E_x = \frac{60i_A}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}} \cos \alpha$$

$$E_y = \frac{60i_A}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}} \sin \alpha$$

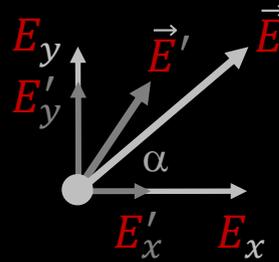
$$\cos \alpha = \frac{x + \frac{\sqrt{3}}{4}d}{r} = \frac{x + \frac{\sqrt{3}}{4}d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$

$$\sin \alpha = \frac{y}{r} = \frac{y}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}}$$

∴

$$E_x = \frac{60i_A \left[x + \frac{\sqrt{3}}{4}d\right]}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}$$

$$E_y = \frac{60i_A y}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2}$$



$$\vec{E}' = \frac{-60i_A}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2}}$$

$$E'_x = \frac{-60i_A}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2}} \cos \alpha'$$

$$E'_y = \frac{-60i_A}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2}} \sin \alpha'$$

$$\cos \alpha' = \frac{x + \frac{\sqrt{3}}{4}d}{r'} = \frac{x + \frac{\sqrt{3}}{4}d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2}}$$

$$\sin \alpha' = \frac{y + 2h}{r'} = \frac{y + 2h}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2}}$$

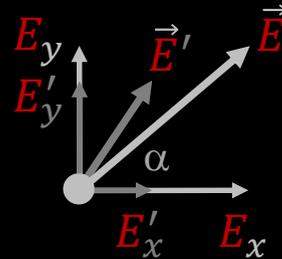
∴

$$E'_x = \frac{-60i_A \left[x + \frac{\sqrt{3}}{4}d\right]}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2}$$

$$E'_y = \frac{-60i_A [y + 2h]}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2}$$

## Calculating 3 $\Phi$ T-Line Electric Fields

final form of the horizontal and vertical components of E field for phase A



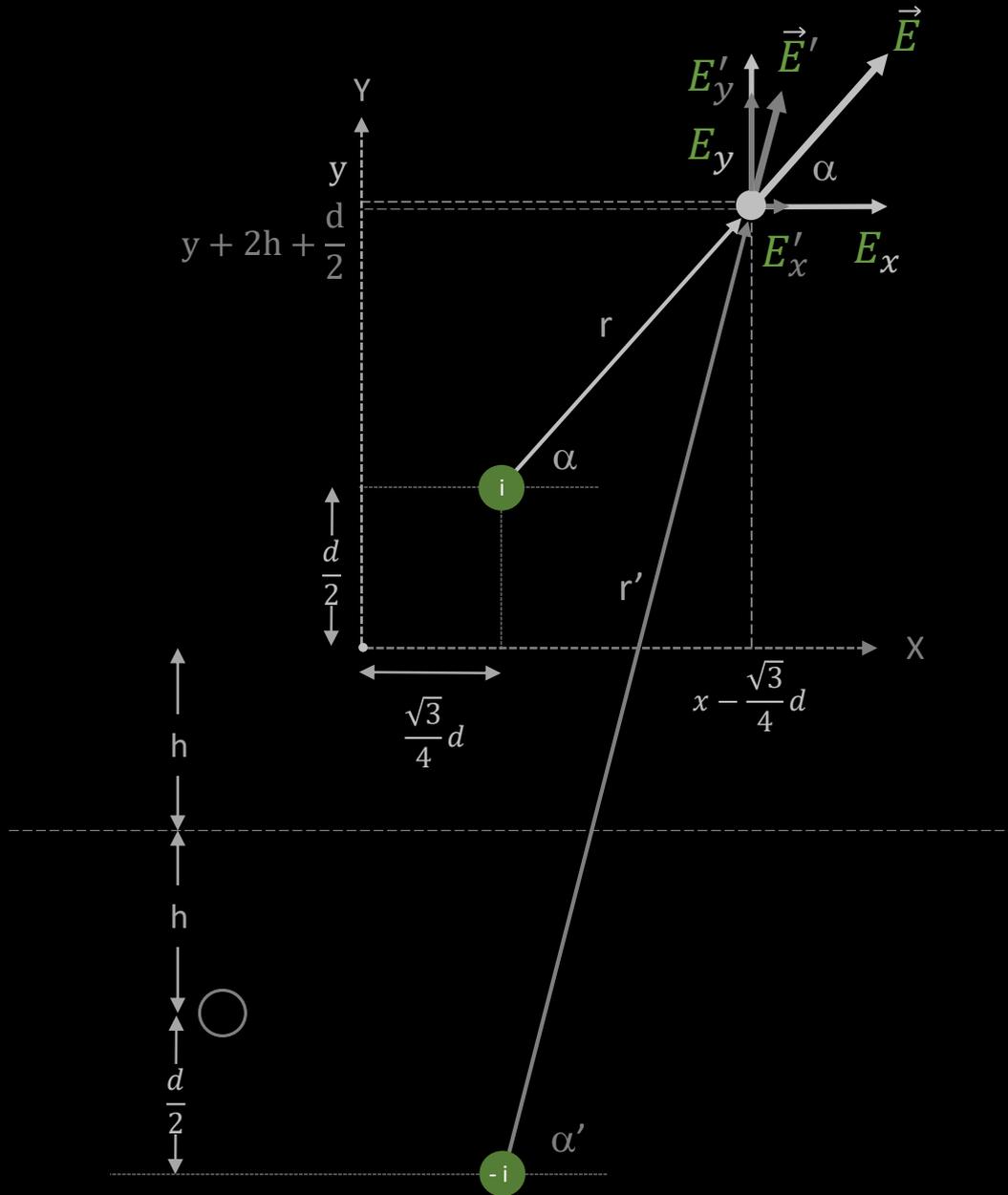
Since this calculation was for the A phase...  
modify the notation slightly and write the total A phase electric field

$$E_{Ax} = \frac{60i_A \left[ x + \frac{\sqrt{3}}{4} d \right]}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + y^2} - \frac{60i_A \left[ x + \frac{\sqrt{3}}{4} d \right]}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h]^2}$$

$$E_{Ay} = \frac{60i_A y}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + y^2} - \frac{60i_A [y + 2h]}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h]^2}$$

# Calculating 3Φ T-Line Electric Fields

repeat for B phase



calculate r

$$r = \sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}$$

calculate r'

$$r' = \sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2}$$

$$\vec{E} = \frac{60i_B}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}$$

$$\vec{E}' = \frac{-60i_B}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2}}$$

## Calculating 3Φ T-Line Electric Fields

get the horizontal and vertical components of E and E' for B phase

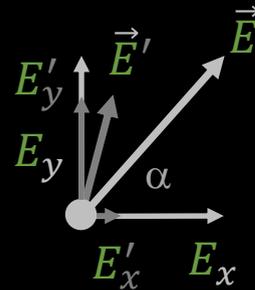
$$\vec{E} = \frac{60i_B}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}$$

$$E_x = \frac{60i_B}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}} \cos \alpha$$

$$E_y = \frac{60i_B}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}} \sin \alpha$$

$$\cos \alpha = \frac{x - \frac{\sqrt{3}}{4}d}{r} = \frac{x - \frac{\sqrt{3}}{4}d}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}$$

$$\sin \alpha = \frac{y - \frac{d}{2}}{r} = \frac{y - \frac{d}{2}}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}}$$



$$\vec{E}' = \frac{-60i_B}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2}}$$

$$E'_x = \frac{-60i_B}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2}} \cos \alpha'$$

$$E'_y = \frac{-60i_B}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2}} \sin \alpha'$$

$$\cos \alpha' = \frac{x - \frac{\sqrt{3}}{4}d}{r'} = \frac{x - \frac{\sqrt{3}}{4}d}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2}}$$

$$\sin \alpha' = \frac{y + 2h + \frac{d}{2}}{r'} = \frac{y + 2h + \frac{d}{2}}{\sqrt{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2}}$$

$$\therefore$$

$$E_x = \frac{60i_B \left[x - \frac{\sqrt{3}}{4}d\right]}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}$$

$$E_y = \frac{60i_B \left[y - \frac{d}{2}\right]}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2}$$

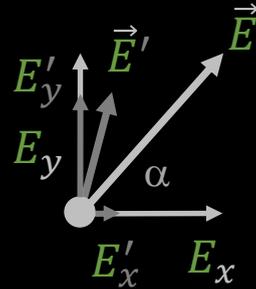
$$\therefore$$

$$E'_x = \frac{-60i_B \left[x - \frac{\sqrt{3}}{4}d\right]}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2}$$

$$E'_y = \frac{-60i_B \left[y + 2h + \frac{d}{2}\right]}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2}$$

## Calculating 3 $\Phi$ T-Line Electric Fields

final form of the horizontal and vertical components of E field for phase B



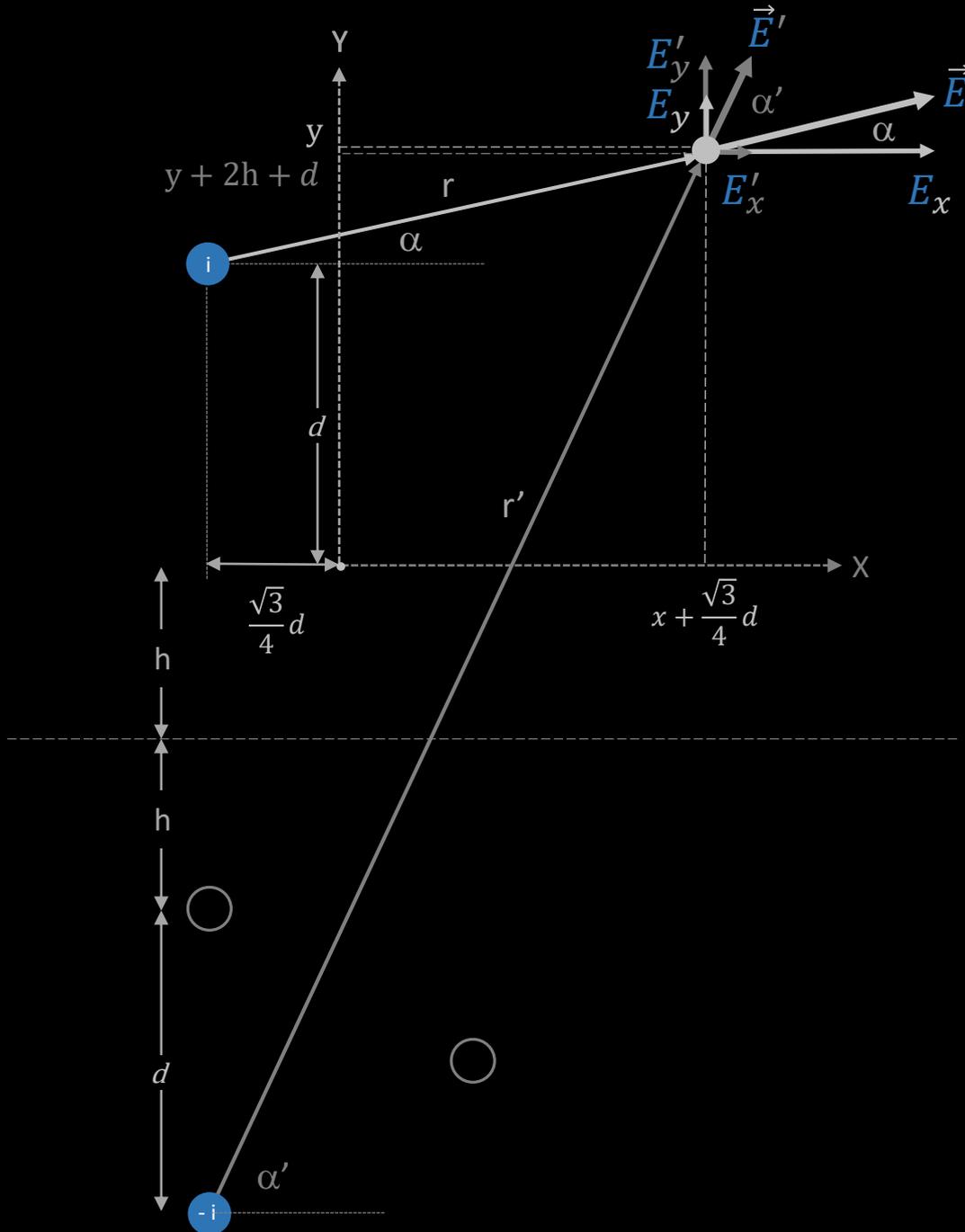
Since this calculation was for the B phase...  
modify the notation slightly and write the total B phase electric field

$$E_{Bx} = \frac{60i_B \left[ x - \frac{\sqrt{3}}{4} d \right]}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y - \frac{d}{2} \right]^2} - \frac{60i_B \left[ x - \frac{\sqrt{3}}{4} d \right]}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y + 2h + \frac{d}{2} \right]^2}$$

$$E_{By} = \frac{60i_B \left[ y - \frac{d}{2} \right]}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y - \frac{d}{2} \right]^2} - \frac{60i_B \left[ y + 2h + \frac{d}{2} \right]}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y + 2h + \frac{d}{2} \right]^2}$$

# Calculating 3Φ T-Line Electric Fields

repeat for C phase



calculate r

$$r = \sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}$$

calculate r'

$$r' = \sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}$$

$$\vec{E} = \frac{60i_C}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}}$$

$$\vec{E}' = \frac{-60i_C}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}}$$

## Calculating 3Φ T-Line Electric Fields

get the horizontal and vertical components of E and E' for C phase

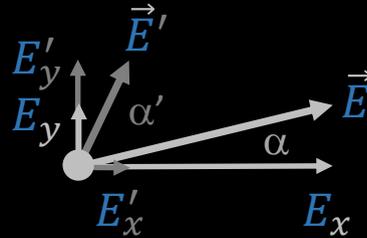
$$\vec{E} = \frac{60i_C}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}}$$

$$E_x = \frac{60i_C}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}} \cos \alpha$$

$$E_y = \frac{60i_C}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}} \sin \alpha$$

$$\cos \alpha = \frac{x + \frac{\sqrt{3}}{4}d}{r} = \frac{x + \frac{\sqrt{3}}{4}d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}}$$

$$\sin \alpha = \frac{y - d}{r} = \frac{y - d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}}$$



$$\vec{E}' = \frac{-60i_C}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}}$$

$$E'_x = \frac{-60i_C}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}} \cos \alpha'$$

$$E'_y = \frac{-60i_C}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}} \sin \alpha'$$

$$\cos \alpha' = \frac{x + \frac{\sqrt{3}}{4}d}{r'} = \frac{x + \frac{\sqrt{3}}{4}d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}}$$

$$\sin \alpha' = \frac{y + 2h + d}{r'} = \frac{y + 2h + d}{\sqrt{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}}$$

∴

$$E_x = \frac{60i_C \left[x + \frac{\sqrt{3}}{4}d\right]}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}$$

$$E_y = \frac{60i_C [y - d]}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}$$

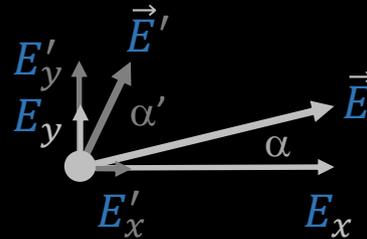
∴

$$E'_x = \frac{-60i_C \left[x + \frac{\sqrt{3}}{4}d\right]}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}$$

$$E'_y = \frac{-60i_C [y + 2h + d]}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}$$

## Calculating 3 $\Phi$ T-Line Electric Fields

final form of the horizontal and vertical components of E field for phase C



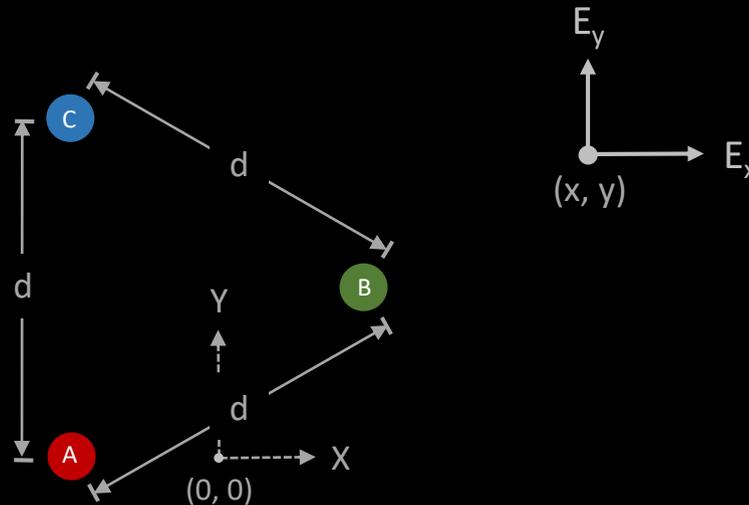
Since this calculation was for the C phase...  
modify the notation slightly and write the total C phase electric field

$$E_{Cx} = \frac{60i_c \left[ x + \frac{\sqrt{3}}{4}d \right]}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y - d]^2} - \frac{60i_c \left[ x + \frac{\sqrt{3}}{4}d \right]}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y + 2h + d]^2}$$

$$E_{Cy} = \frac{60i_c [y - d]}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y - d]^2} - \frac{60i_c [y + 2h + d]}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y + 2h + d]^2}$$

## Calculating 3Φ T-Line Electric Fields

so far we have the E field at any arbitrary point at single instant in time...



### Substitute the Time Dependent AC Current for each Phase

for a balanced 3Φ system in positive sequence...  
 all three phases have the same RMS current amplitude =  $I$   
 rotating counter clockwise  
 and are out of phase by  $120^\circ$

$$i_A = I \cos(\omega t)$$

$$i_B = I \cos(\omega t - 120^\circ)$$

$$i_C = I \cos(\omega t + 120^\circ)$$

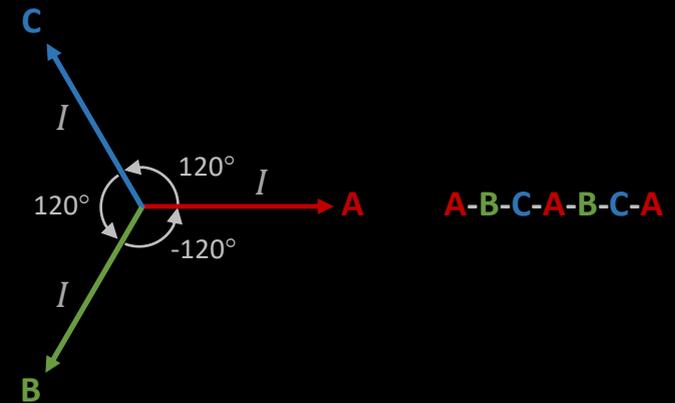
$$\omega = 2\pi f = \theta$$

$$i_A = I \cos(\theta)$$

$$i_B = I \cos(\theta - 120^\circ)$$

$$i_C = I \cos(\theta + 120^\circ)$$

$$\omega = 2\pi f$$



# Calculating 3Φ T-Line Electric Fields

substitute alternating phase currents

$$i_A = I \cos \theta_A \quad i_B = I \cos \theta_B \quad i_C = I \cos \theta_C$$

## Horizontal Components

$$E_{Ax} = 60I \left[ \frac{\left[ x + \frac{\sqrt{3}}{4} d \right] \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + y^2} - \frac{\left[ x + \frac{\sqrt{3}}{4} d \right] \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h]^2} \right]$$
$$E_{Bx} = 60I \left[ \frac{\left[ x - \frac{\sqrt{3}}{4} d \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y - \frac{d}{2} \right]^2} - \frac{\left[ x - \frac{\sqrt{3}}{4} d \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y + 2h + \frac{d}{2} \right]^2} \right]$$
$$E_{Cx} = 60I \left[ \frac{\left[ x + \frac{\sqrt{3}}{4} d \right] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y - d]^2} - \frac{\left[ x + \frac{\sqrt{3}}{4} d \right] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h + d]^2} \right]$$

## Vertical Components

$$E_{Ay} = 60I \left[ \frac{y \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + y^2} - \frac{[y + 2h] \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h]^2} \right]$$
$$E_{By} = 60I \left[ \frac{\left[ y - \frac{d}{2} \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y - \frac{d}{2} \right]^2} - \frac{\left[ y + 2h + \frac{d}{2} \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y + 2h + \frac{d}{2} \right]^2} \right]$$
$$E_{Cy} = 60I \left[ \frac{[y - d] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y - d]^2} - \frac{[y + 2h + d] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h + d]^2} \right]$$

## Calculating 3Φ T-Line Electric Fields

now add A B C phase contributions together to get total E field at any point (x, y) and any time

$$E_x = E_{Ax} + E_{Bx} + E_{Cx}$$

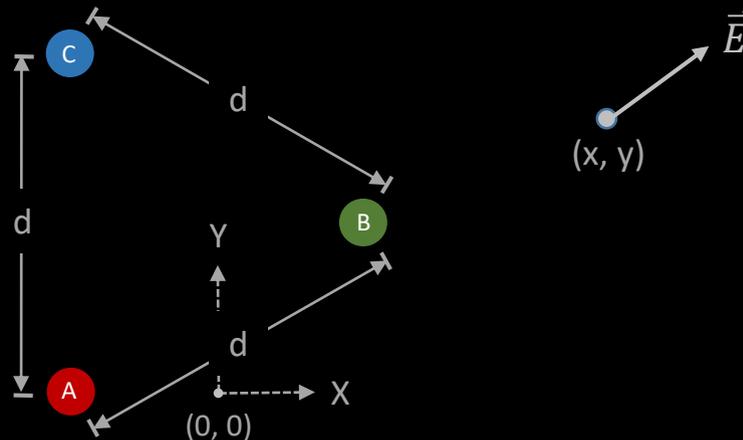
$$E_x = 60I \left[ \frac{\left[ x + \frac{\sqrt{3}}{4} d \right] \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + y^2} + \frac{\left[ x - \frac{\sqrt{3}}{4} d \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y - \frac{d}{2} \right]^2} + \frac{\left[ x + \frac{\sqrt{3}}{4} d \right] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y - d]^2} - \frac{\left[ x + \frac{\sqrt{3}}{4} d \right] \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h]^2} - \frac{\left[ x - \frac{\sqrt{3}}{4} d \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y + 2h + \frac{d}{2} \right]^2} - \frac{\left[ x + \frac{\sqrt{3}}{4} d \right] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h + d]^2} \right]$$

$$E_y = E_{Ay} + E_{By} + E_{Cy}$$

$$E_y = 60I \left[ \frac{y \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + y^2} + \frac{\left[ y - \frac{d}{2} \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y - \frac{d}{2} \right]^2} + \frac{[y - d] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y - d]^2} - \frac{[y + 2h] \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h]^2} - \frac{\left[ y + 2h + \frac{d}{2} \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4} d \right]^2 + \left[ y + 2h + \frac{d}{2} \right]^2} - \frac{[y + 2h + d] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4} d \right]^2 + [y + 2h + d]^2} \right]$$

image conductors contribution

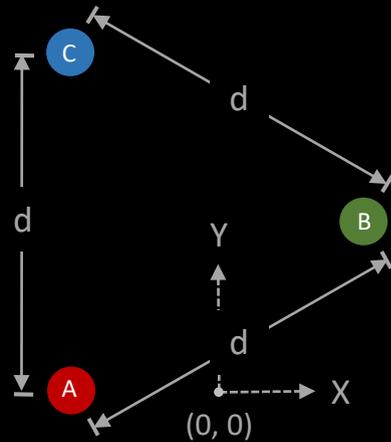
$$\vec{E} = E_x \hat{x} + E_y \hat{y}$$



You could plot the E field vectors for any number of points in space for any instant in time. The length of the vector would indicate the electric field magnitude. The direction of the vector would indicate if the field is positive or negative. This method of illustrating fields is not very exciting.... Let's consider another way

# Animating 3Φ T-Line Electric Fields

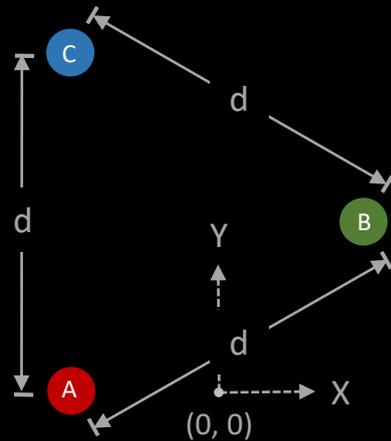
For this exercise we would like to animate the magnitude of the E field =  $|E|$

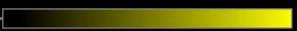


  $(x, y)$  ← maybe use shades of yellow to illustrate E field intensity

$$|E| = \sqrt{E_x^2 + E_y^2}$$

The above method would be a good start but it would not give an indication of the polarity or direction of the field. We want to illustrate positive field intensity with one color and negative field intensity with another color



  $(x, y)$  ← maybe use shades of yellow to illustrate positive E field intensity

 ← and shades of green to illustrate negative E field intensity

## Animating 3Φ T-Line Electric Fields

It would seem that we have all the information we need to determine the polarity, But ...  
the E field magnitude has no sign.

If there were only one conductor...

we could use the sign or direction of the current.

We need to:

- 1) find the contribution (with sign) from each phase
- 2) use superposition to add up the contributions
- 3) determine if the net contribution is positive or negative

Returning to the previous derivation...

and using the **A** phase conductor as an example:

We could use the product of the magnitude of **E** and  $\pm 1$   
(depending on current direction) to determine the  
contribution of the E field to polarity or direction.

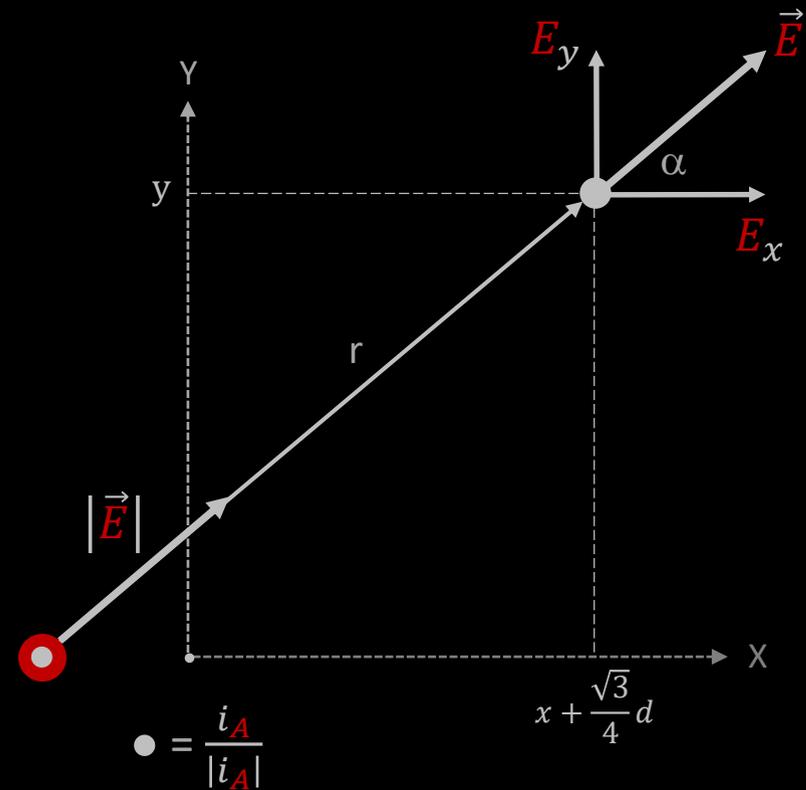
we can get  $\pm 1$  with  $\frac{i_A}{|i_A|}$

let  $\delta_A =$  contribution of  $\vec{E}$  to polarity or direction

$$\therefore \delta_A = |\vec{E}| \frac{i_A}{|i_A|}$$

Since this calculation was for the center **A** phase...  
modify the notation slightly:

$$\delta_A = |\vec{E}_A| \frac{i_A}{|i_A|}$$

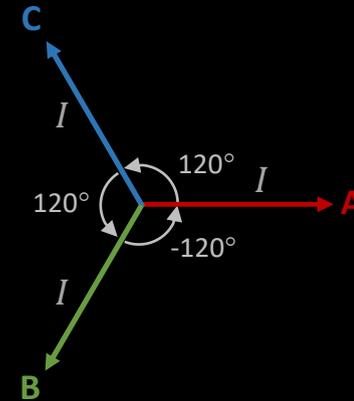


the opposite and very small contribution  
of the image can be ignored

## Animating 3Φ T-Line Electric Fields

continue with the sign or polarity of **A** and **C** phase

$$\begin{aligned}
 i_A &= I \cos[\theta] & \theta_A &= \theta & i_A &= I \cos \theta_A \\
 i_B &= I \cos\left[\theta - \frac{2\pi}{3}\right] & \theta_B &= \theta - \frac{2\pi}{3} & i_B &= I \cos \theta_B \\
 i_C &= I \cos\left[\theta + \frac{2\pi}{3}\right] & \theta_C &= \theta + \frac{2\pi}{3} & i_C &= I \cos \theta_C \\
 \theta &= \omega t = 2\pi f t & & & \theta &= \omega t
 \end{aligned}$$



substituting **ABC** alternating phase currents...

$$\delta_A = |\vec{E}_A| \frac{i_A}{|i_A|} = |\vec{E}_A| \frac{I \cos \theta_A}{|I \cos \theta_A|} = |\vec{E}_A| \frac{\cos \theta_A}{|\cos \theta_A|}$$

$$\delta_B = |\vec{E}_B| \frac{i_B}{|i_B|} = |\vec{E}_B| \frac{I \cos \theta_B}{|I \cos \theta_B|} = |\vec{E}_B| \frac{\cos \theta_B}{|\cos \theta_B|}$$

$$\delta_C = |\vec{E}_C| \frac{i_C}{|i_C|} = |\vec{E}_C| \frac{I \cos \theta_C}{|I \cos \theta_C|} = |\vec{E}_C| \frac{\cos \theta_C}{|\cos \theta_C|}$$

$$\delta_A = |\vec{E}_A| \frac{\cos \theta_A}{|\cos \theta_A|} \quad \delta_B = |\vec{E}_B| \frac{\cos \theta_B}{|\cos \theta_B|} \quad \delta_C = |\vec{E}_C| \frac{\cos \theta_C}{|\cos \theta_C|}$$

# Animating 3Φ T-Line Electric Fields

get the total contribution to the E field

$$\delta_A = |\vec{E}_A| \frac{\cos \theta_A}{|\cos \theta_A|} \quad \delta_B = |\vec{E}_B| \frac{\cos \theta_B}{|\cos \theta_B|} \quad \delta_C = |\vec{E}_C| \frac{\cos \theta_C}{|\cos \theta_C|}$$

total contribution to the resulting  $\vec{E}$  field =  $\delta_A + \delta_B + \delta_C$

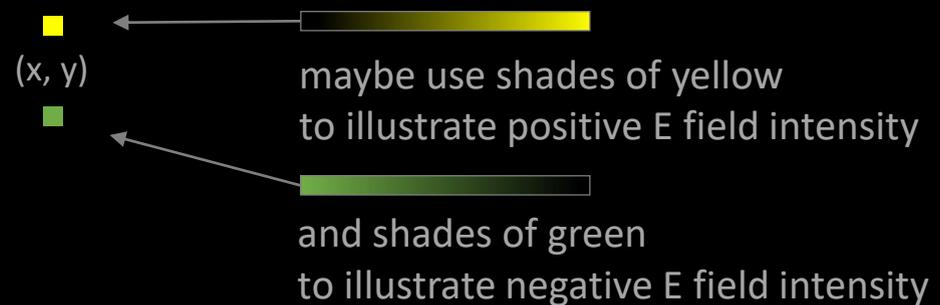
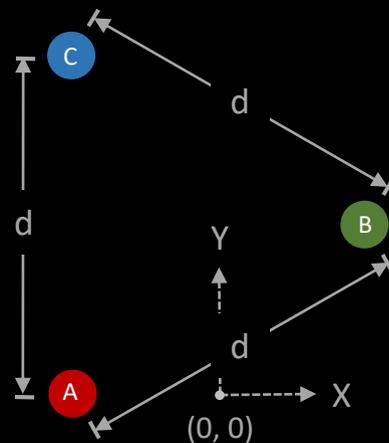
but we are only interested in the sign ( $\pm 1$ ) of the total contribution

the sign ( $\pm 1$ ) of the total contribution to the resulting  $\vec{E}$  field =  $\frac{\delta_A + \delta_B + \delta_C}{|\delta_A + \delta_B + \delta_C|}$

now we can add the polarity or direction to the magnitude of the E field

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} \quad \vec{E}_{polarity} = \frac{\delta_A + \delta_B + \delta_C}{|\delta_A + \delta_B + \delta_C|}$$

we can now plot  $E(x, y, d, h, \theta) = \frac{\delta_A + \delta_B + \delta_C}{|\delta_A + \delta_B + \delta_C|} \sqrt{E_x^2 + E_y^2}$



$$E = \frac{\delta_A + \delta_B + \delta_C}{|\delta_A + \delta_B + \delta_C|} \sqrt{E_x^2 + E_y^2}$$

# Summary So Far

$$E_x = 60I \left[ \frac{\left[ x + \frac{\sqrt{3}}{4}d \right] \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + y^2} + \frac{\left[ x - \frac{\sqrt{3}}{4}d \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4}d \right]^2 + \left[ y - \frac{d}{2} \right]^2} + \frac{\left[ x + \frac{\sqrt{3}}{4}d \right] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y - d]^2} - \frac{\left[ x + \frac{\sqrt{3}}{4}d \right] \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y + 2h]^2} - \frac{\left[ x - \frac{\sqrt{3}}{4}d \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4}d \right]^2 + \left[ y + 2h + \frac{d}{2} \right]^2} - \frac{\left[ x + \frac{\sqrt{3}}{4}d \right] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y + 2h + d]^2} \right]$$

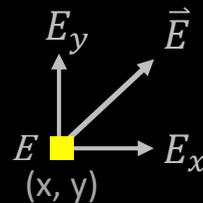
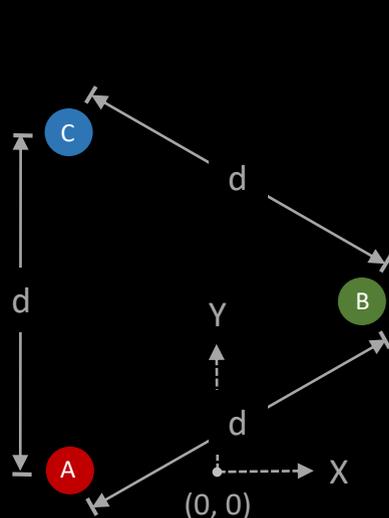
drop and normalize

$$E_x = 60I \left[ \frac{E_{Ax}}{E_{Ax}^2 + y^2} + \frac{E_{Bx}}{E_{Bx}^2 + \left[ y - \frac{d}{2} \right]^2} + \frac{E_{Cx}}{E_{Cx}^2 + [y - d]^2} - \frac{E'_{Ax}}{E_{Ax}^2 + [y + 2h]^2} - \frac{E'_{Bx}}{E_{Bx}^2 + \left[ y + 2h + \frac{d}{2} \right]^2} - \frac{E'_{Cx}}{E_{Cx}^2 + [y + 2h + d]^2} \right]$$

$$E_y = 60I \left[ \frac{y \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + y^2} + \frac{\left[ y - \frac{d}{2} \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4}d \right]^2 + \left[ y - \frac{d}{2} \right]^2} + \frac{[y - d] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y - d]^2} - \frac{[y + 2h] \cos \theta_A}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y + 2h]^2} - \frac{\left[ y + 2h + \frac{d}{2} \right] \cos \theta_B}{\left[ x - \frac{\sqrt{3}}{4}d \right]^2 + \left[ y + 2h + \frac{d}{2} \right]^2} - \frac{[y + 2h + d] \cos \theta_C}{\left[ x + \frac{\sqrt{3}}{4}d \right]^2 + [y + 2h + d]^2} \right]$$

$$E_y = 60I \left[ \frac{E_{Ay}}{E_{Ax}^2 + y^2} + \frac{E_{By}}{E_{Bx}^2 + \left[ y - \frac{d}{2} \right]^2} + \frac{E_{Cy}}{E_{Cx}^2 + [y - d]^2} - \frac{E'_{Ay}}{E_{Ax}^2 + [y + 2h]^2} - \frac{E'_{By}}{E_{Bx}^2 + \left[ y + 2h + \frac{d}{2} \right]^2} - \frac{E'_{Cy}}{E_{Cx}^2 + [y + 2h + d]^2} \right]$$

$$\delta_A = |\vec{E}_A| \frac{\cos \theta_A}{|\cos \theta_A|} \quad \delta_B = |\vec{E}_B| \frac{\cos \theta_B}{|\cos \theta_B|} \quad \delta_C = |\vec{E}_C| \frac{\cos \theta_C}{|\cos \theta_C|}$$



$$E = \frac{\delta_A + \delta_B + \delta_C}{|\delta_A + \delta_B + \delta_C|} \sqrt{\frac{E_x^2 + E_y^2}{E_{Base}}}$$

now lets find the "Base" to normalize on

# Animating 3Φ T-Line Electric Fields

find the maximum E field for normalizing

consider **A** phase:

assume  $\theta = 0^\circ$  (max **A** phase current)

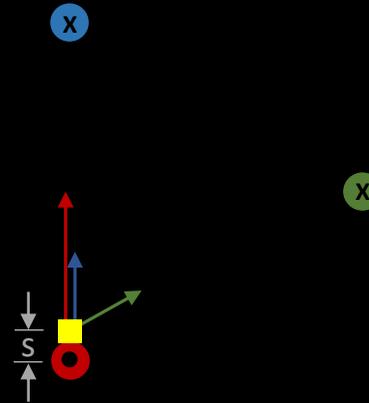
$x = -\frac{\sqrt{3}}{4}d$  at the center of the A phase conductor.

take a very small step in the  $y$  direction  
(on the order of conductor radius, only inches!)

call this very small step =  $s$

the  $x$  coordinate of interest is:  $x = -\frac{\sqrt{3}}{4}d$

the  $y$  coordinate of interest is:  $y = s$



$$E_x = \frac{\left[x + \frac{\sqrt{3}}{4}d\right] \cos \theta_A}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2} + \frac{\left[x - \frac{\sqrt{3}}{4}d\right] \cos \theta_B}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2} + \frac{\left[x + \frac{\sqrt{3}}{4}d\right] \cos \theta_C}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2} - \frac{\left[x + \frac{\sqrt{3}}{4}d\right] \cos \theta_A}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2} - \frac{\left[x - \frac{\sqrt{3}}{4}d\right] \cos \theta_B}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2} - \frac{\left[x + \frac{\sqrt{3}}{4}d\right] \cos \theta_C}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}$$

ignore contribution from image conductors

$$E_x \approx \frac{\left[x + \frac{\sqrt{3}}{4}d\right] \cos \theta_A}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2} + \frac{\left[x - \frac{\sqrt{3}}{4}d\right] \cos \theta_B}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2} + \frac{\left[x + \frac{\sqrt{3}}{4}d\right] \cos \theta_C}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}$$

$$E_y = \frac{y \cos \theta_A}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2} + \frac{\left[y - \frac{d}{2}\right] \cos \theta_B}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2} + \frac{[y - d] \cos \theta_C}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2} - \frac{[y + 2h] \cos \theta_A}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h]^2} - \frac{\left[y + 2h + \frac{d}{2}\right] \cos \theta_B}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y + 2h + \frac{d}{2}\right]^2} - \frac{[y + 2h + d] \cos \theta_C}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y + 2h + d]^2}$$

ignore contribution from image conductors

$$E_y \approx \frac{y \cos \theta_A}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2} + \frac{\left[y - \frac{d}{2}\right] \cos \theta_B}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2} + \frac{[y - d] \cos \theta_C}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}$$

# Animating 3Φ T-Line Electric Fields

find the maximum E field for normalizing

$$E_x = \frac{\left[x + \frac{\sqrt{3}}{4}d\right] \cos \theta_A}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2} + \frac{\left[x - \frac{\sqrt{3}}{4}d\right] \cos \theta_B}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2} + \frac{\left[x + \frac{\sqrt{3}}{4}d\right] \cos \theta_C}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}$$

$$E_x = \frac{\left[-\frac{\sqrt{3}}{4}d + \frac{\sqrt{3}}{4}d\right]}{\left[-\frac{\sqrt{3}}{4}d + \frac{\sqrt{3}}{4}d\right]^2 + s^2} + \frac{-0.5\left[-\frac{\sqrt{3}}{4}d - \frac{\sqrt{3}}{4}d\right]}{\left[-\frac{\sqrt{3}}{4}d - \frac{\sqrt{3}}{4}d\right]^2 + \left[s - \frac{d}{2}\right]^2} + \frac{-0.5\left[-\frac{\sqrt{3}}{4}d + \frac{\sqrt{3}}{4}d\right]}{\left[-\frac{\sqrt{3}}{4}d + \frac{\sqrt{3}}{4}d\right]^2 + [s - d]^2}$$

$$E_x = \frac{\frac{\sqrt{3}}{4}d}{\left[\frac{\sqrt{3}}{2}d\right]^2 + \left[s - \frac{d}{2}\right]^2} = \frac{\frac{\sqrt{3}}{4}d}{\frac{3}{4}d^2 + s^2 - sd + \frac{d^2}{4}}$$

$$E_x = \frac{\frac{\sqrt{3}}{4}d}{d^2 - s[d - s]} \quad \leftarrow \text{Max Horizontal Component at } \omega t = 0$$

plug in:

$$x = -\frac{\sqrt{3}}{4}d$$

$$y = s$$

$$\cos \theta_A = 1.0$$

$$\cos \theta_B = -0.5$$

$$\cos \theta_C = -0.5$$

$$E_{Base} = |E|_{max} = \sqrt{E_x^2 + E_y^2}$$

“Base” Electric Field for normalizing  
(numeric solution)

$$E_y = \frac{y \cos \theta_A}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + y^2} + \frac{\left[y - \frac{d}{2}\right] \cos \theta_B}{\left[x - \frac{\sqrt{3}}{4}d\right]^2 + \left[y - \frac{d}{2}\right]^2} + \frac{[y - d] \cos \theta_C}{\left[x + \frac{\sqrt{3}}{4}d\right]^2 + [y - d]^2}$$

$$E_y = \frac{s}{\left[-\frac{\sqrt{3}}{4}d + \frac{\sqrt{3}}{4}d\right]^2 + s^2} + \frac{-0.5\left[s - \frac{d}{2}\right]}{\left[-\frac{\sqrt{3}}{4}d - \frac{\sqrt{3}}{4}d\right]^2 + \left[s - \frac{d}{2}\right]^2} + \frac{-0.5[s - d]}{\left[-\frac{\sqrt{3}}{4}d + \frac{\sqrt{3}}{4}d\right]^2 + [s - d]^2}$$

$$E_y = \frac{1}{s} - \frac{0.5\left[s - \frac{d}{2}\right]}{\left[s - \frac{d}{2}\right]^2} - \frac{0.5[s - d]}{[s - d]^2} = \frac{1}{s} - \frac{0.5}{s - \frac{d}{2}} - \frac{0.5}{s - d}$$

$$E_y = \frac{1}{s} + \frac{1}{d - 2s} + \frac{1}{2[d - s]} \quad \leftarrow \text{Max Vertical Component at } \omega t = 0$$

## pseudo code for animating T-Line Electric Fields

```
d = 20.0
h = 60.0
Ebase = f(d,s)

for angle in range(0,361)
    for x in range(-40,40,0.2)
        for y in range(-40,60,0.2)
            angle_a = angle
            angle_b = angle_a-120
            angle_c = angle_a+120
            Eax = f(x,y,d,h,angle_a)
            Ebx = f(x,y,d,h,angle_b)
            Ecx = f(x,y,d,h,angle_c)
            Eay = f(x,y,d,h,angle_a)
            Eby = f(x,y,d,h,angle_b)
            Ecy = f(x,y,d,h,angle_c)
            |Ea| = pow(Eax2+Eay2,0.5)
            |Eb| = pow(Ebx2+Eby2,0.5)
            |Ec| = pow(Ecx2+Ecy2,0.5)
            da = |Ea|*cos(angle_a)/abs(cos(angle_a))
            db = |Eb|*cos(angle_b)/abs(cos(angle_b))
            dc = |Ec|*cos(angle_c)/abs(cos(angle_c))
            polarity = (da+db+dc)/abs(da+db+dc)
            Ex = Bax+Bbx+Bcx
            Ey = Bay+Bby+Bcy
            |E| = pow(Ex2+Ey2,0.5)
            B = polarity*|E|/Ebase

# define conductor spacing
# define height above ground of lowest phase
# calculate the max field (for normalizing)

# loop current angles (phase A is reference)
# loop x coordinates from -2*d to 2*d in steps of d/100
# loop y coordinates from -2*d to 3*d in steps of d/100
# get phase A angle (phase A is reference)
# get phase B angle
# get phase C angle
# calculate A phase field in x direction
# calculate B phase field in x direction
# calculate C phase field in x direction
# calculate A phase field in y direction
# calculate B phase field in y direction
# calculate C phase field in y direction
# calculate magnitude A phase field
# calculate magnitude B phase field
# calculate magnitude C phase field
# calculate A phase contribution
# calculate B phase contribution
# calculate C phase contribution
# get the sign of the magnetic field
# get net field in x direction
# get net field in y direction
# get magnitude of net field
# report normalized field magnitude, with sign

# this nested loop will give you the field at one (x,y) coordinate at one current angle
# we recommend looking into matplotlib and seaborn for generating xy grid and using heatmaps
```



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Questions or Comments ...

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