Compensated Bus Voltage Change Derivation

$$\overrightarrow{MVA}_{SC} = V_S \vec{I}_{SC}^* \qquad \vec{I}_{SC} = \frac{V_S}{Z_S} \qquad \overrightarrow{MVA}_{SC} = \frac{|V_S|^2}{\vec{Z}_S^*} \qquad \vec{Z}_S^* = \frac{|V_S|^2}{\overrightarrow{MVA}_{SC}}$$

$$\vec{S} = \vec{V}_B \vec{I}^* \qquad \vec{I}^* = \frac{\vec{S}}{\vec{V}_B}$$

$$I = \frac{V_S - \vec{V}_B}{\vec{Z}_S} \qquad \vec{I}^* = \frac{V_S - \vec{V}_B^*}{\vec{Z}_S^*}$$

$$\frac{\vec{S}}{\vec{V}_B} = \frac{V_S - \vec{V}_B^*}{\vec{Z}_S^*}$$

$$\vec{S}\vec{Z}_S^* = V_S \vec{V}_B - \vec{V}_B^* \vec{V}_B$$

$$|V_B|^2 - V_S \vec{V}_B + \vec{S}\vec{Z}_S^* = 0$$

$$|V_B|^2 - V_S \vec{V}_B + \frac{\vec{S}|V_S|^2}{\overrightarrow{MVA}_{SC}} = 0$$
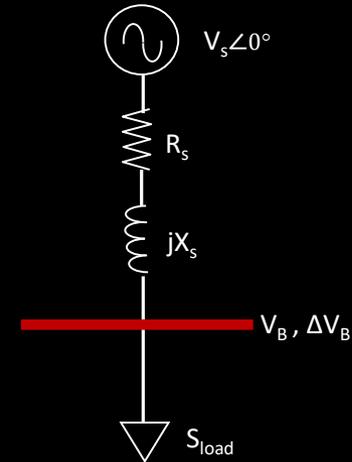
$$Assume: |V_S|^2 \approx |V_B|^2$$

$$|V_B|^2 - V_S \vec{V}_B + \frac{\vec{S}|V_B|^2}{\overrightarrow{MVA}_{SC}} = 0 \qquad Let\ \vec{A} = \frac{\vec{S}}{\overrightarrow{MVA}_{SC}}$$

$$|V_B|^2 + \vec{A}|V_B|^2 - V_S|V_B|e^{j\theta_B} = 0$$

$$(\vec{A} + 1)|V_B|^2 - V_S|V_B|e^{j\theta_B} = 0$$

$$Assume: \theta_B \approx \theta_S = 0$$

$$(\vec{A} + 1)|V_B|^2 - V_S|V_B| = 0$$

$$use\ quadradic\ form:$$

$$V_B = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

V$_s$∠0°

R$_s$

jX$_s$

V$_B$, ΔV$_B$

S$_{load}$

$$V_B = \frac{V_S \pm \sqrt{V_S^2 - 4(\vec{A}+1)0}}{2(\vec{A}+1)} = \frac{V_S \pm V_S}{2(\vec{A}+1)} = \frac{2V_S}{2(\vec{A}+1)} = \frac{V_S}{(\vec{A}+1)}$$

$$\vec{V}_B = \frac{V_S}{\vec{A}+1}$$

$$\frac{\overrightarrow{\Delta V_B}}{\vec{V}_{B0}} = \frac{\vec{V}_{Bf} - \vec{V}_{B0}}{\vec{V}_{B0}}$$

$$\vec{V}_{B0} = \frac{V_S}{\vec{A}_0 + 1} \qquad \vec{V}_{Bf} = \frac{V_S}{\vec{A}_f + 1}$$

$$\frac{\overrightarrow{\Delta V_B}}{\vec{V}_{B0}} = \frac{\dfrac{V_S}{\vec{A}_f + 1} - \dfrac{V_S}{\vec{A}_0 + 1}}{\dfrac{V_S}{\vec{A}_0 + 1}} = \frac{\vec{A}_0 + 1}{\vec{A}_f + 1} - 1 = \frac{\vec{A}_0 - \vec{A}_f}{\vec{A}_f + 1}$$

$$\vec{A}_0 = \frac{\vec{S}_0}{\overrightarrow{MVA}_{SC}} \qquad \vec{A}_f = \frac{\vec{S}_f}{\overrightarrow{MVA}_{SC}}$$

$$\frac{\overrightarrow{\Delta V_B}}{\vec{V}_{B0}} = \frac{\dfrac{\vec{S}_0}{\overrightarrow{MVA}_{SC}} - \dfrac{\vec{S}_f}{\overrightarrow{MVA}_{SC}}}{\dfrac{\vec{S}_f}{\overrightarrow{MVA}_{SC}} + 1} = \frac{\vec{S}_0 - \vec{S}_f}{\vec{S}_f + \overrightarrow{MVA}_{SC}}$$

Q is positive if load is inductive
Q is negative if load is capacitive

$$\vec{S}_0 = P + jQ$$
$$\vec{S}_f = P + jQ + j\Delta Q$$

ΔQ is positive if adding inductor
ΔQ is negative if adding capacitor

$$\frac{\overrightarrow{\Delta V_B}}{\vec{V}_{B0}} = \frac{P + jQ - (P + jQ + j\Delta Q)}{P + jQ + j\Delta Q + \overrightarrow{MVA}_{SC}} = \frac{-j\Delta Q}{P + jQ + j\Delta Q + \overrightarrow{MVA}_{SC}}$$

$$\frac{\overrightarrow{\Delta V_B}}{\overrightarrow{V_{B0}}} = \frac{-j\Delta Q}{P + jQ + j\Delta Q + \overrightarrow{MVA_{SC}}}$$

$$if \ X_s \gg R_s:$$

$$\overrightarrow{MVA_{SC}} = j|MVA_{SC}|$$

$$\frac{\overrightarrow{\Delta V_B}}{\overrightarrow{V_{B0}}} = \frac{-j\Delta Q}{P + jQ + j\Delta Q + j|MVA_{SC}|} = \frac{-j\Delta Q}{P + j(|MVA_{SC}| + Q + \Delta Q)}$$

$$\frac{|\overrightarrow{\Delta V_B}|}{|\overrightarrow{V_{B0}}|} = \frac{-\Delta Q}{\sqrt{P^2 + (MVA_{SC} + Q + \Delta Q)^2}}$$

$$if \ MVA_{SC} \gg P_{load}, Q_{load}$$

$$\frac{|\overrightarrow{\Delta V_B}|}{|\overrightarrow{V_{B0}}|} \approx \frac{-\Delta Q}{MVA_{SC}}$$

> Q is positive if load is inductive
> Q is negative if load is capacitive

> ΔQ is positive if adding inductor
> ΔQ is negative if adding capacitor

Summary

This derivation reveals that the amount of voltage change due to switching reactive compensation is dependent on system strength. Keep in mind that during outage conditions, the system strength could be greatly reduced, resulting in much larger change in voltage.

$$\left|\frac{\overrightarrow{\Delta V_B}}{\overrightarrow{V_{B0}}}\right| = \frac{-\Delta Q}{\sqrt{P^2 + (MVA_{SC} + Q + \Delta Q)^2}}$$

This one is actually a very good approximation.
System voltage magnitude and thevenin impedance would be needed for an exact solution of a very complex equation.

$$\left|\frac{\overrightarrow{\Delta V_B}}{\overrightarrow{V_{B0}}}\right| \approx \frac{-\Delta Q}{MVA_{SC}}$$

This one is a very common approximation.
Assumes that the load MVA is much less than the short circuit MVA, which is usually true.

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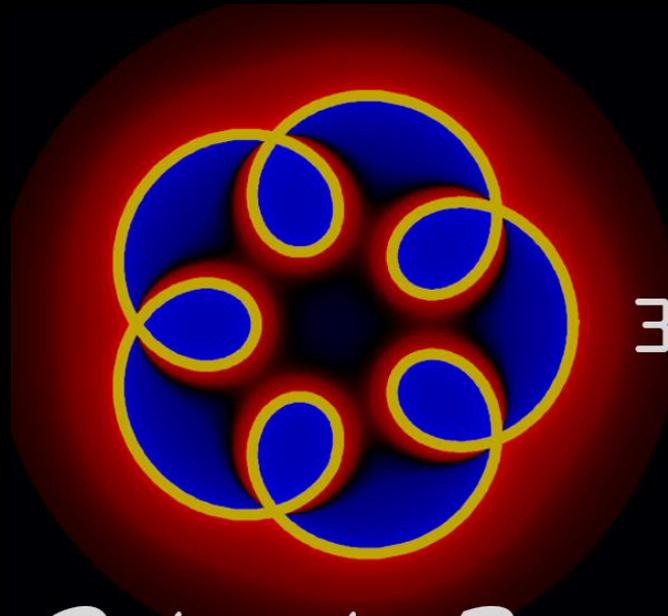
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