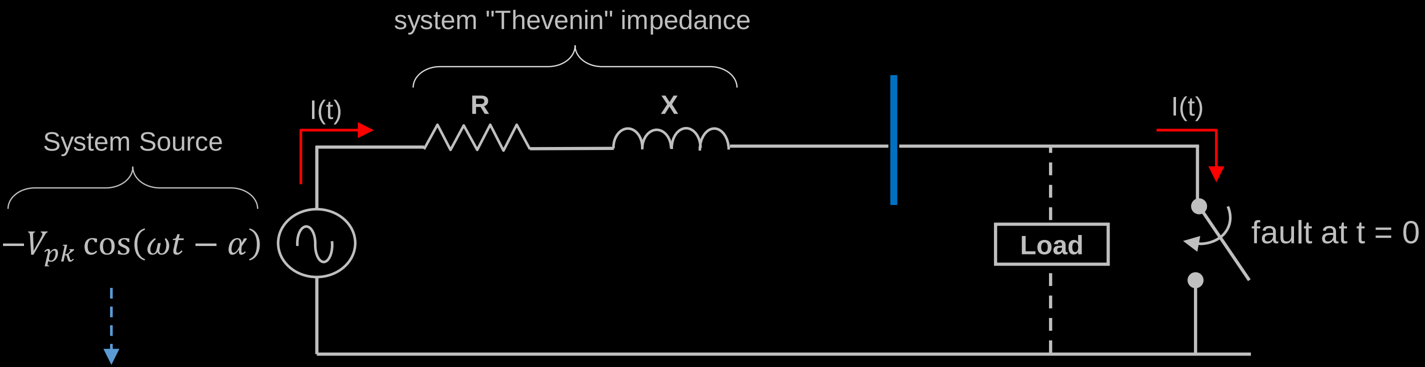
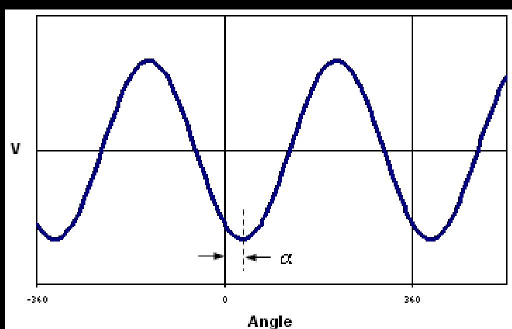
Transient Fault Current Derivation

$$V(t) = -V_{pk}\cos(\omega t - \alpha)$$



load current
is insignificant



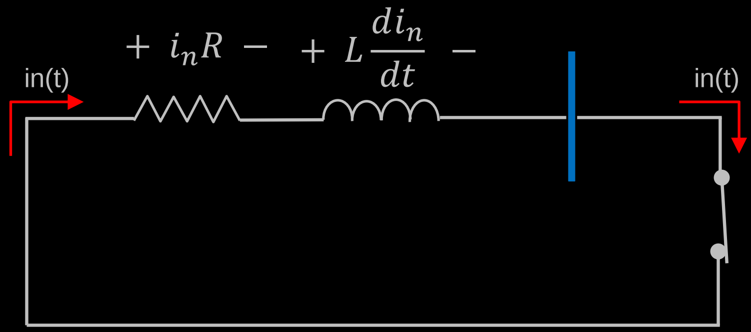
two (2) part solution

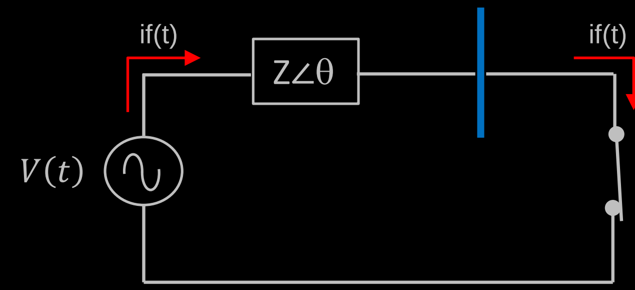
natural response
(differential equations)

$$V_L = L\frac{di}{dt}$$
$$V_R = iR$$

forced response
(simple steady analysis)

$$i = \frac{v}{Z}$$





KVL:

$$i_nR + L\frac{di_n}{dt} = 0$$

$use\ LaPlace$:

$$IR + L(sI + c) = 0$$

$$IR + sLI + cL = 0$$

$$I(R + sL) = -cL$$

$$I = \frac{-c}{s + \frac{R}{L}}$$

$inverse\ LaPlace$:

$$i_n(t) = -ce^{-\frac{R}{L}t}$$

$$i_f(t) = \frac{V(t)}{Z\angle\theta}$$

$$i_f(t) = \frac{-V_{pk}\cos(\omega t - \alpha)}{Z\angle\theta}$$

$$i_f(t) = \frac{-V_{pk}}{|Z|}\cos(\omega t - \alpha - \theta)$$

total response =
natural response + forced response

$$i(t) = -ce^{-\frac{R}{L}t} - \frac{V_{pk}}{|Z|}\cos(\omega t - \alpha - \theta)$$

$$i(t) = -ce^{-\frac{R}{L}t} - \frac{V_{pk}}{|Z|}\cos(\omega t - \alpha - \theta)$$

Fault current at t=0 is zero since inductor can't change current instantaneously

$$i(0) = -c - \frac{V_{pk}}{|Z|}\cos(-\alpha - \theta) = 0$$

$$c = -\frac{V_{pk}}{|Z|}\cos(-\alpha - \theta)$$

$$\mathrm{c} = -\frac{V_{pk}}{|Z|}\cos(\alpha + \theta)$$

$$i(t) = \underbrace{\frac{V_{pk}}{|Z|}\cos(\alpha + \theta)\, e^{-\frac{R}{L}t}}_{} - \underbrace{\frac{V_{pk}}{|Z|}\cos(\omega t - \alpha - \theta)}_{}$$

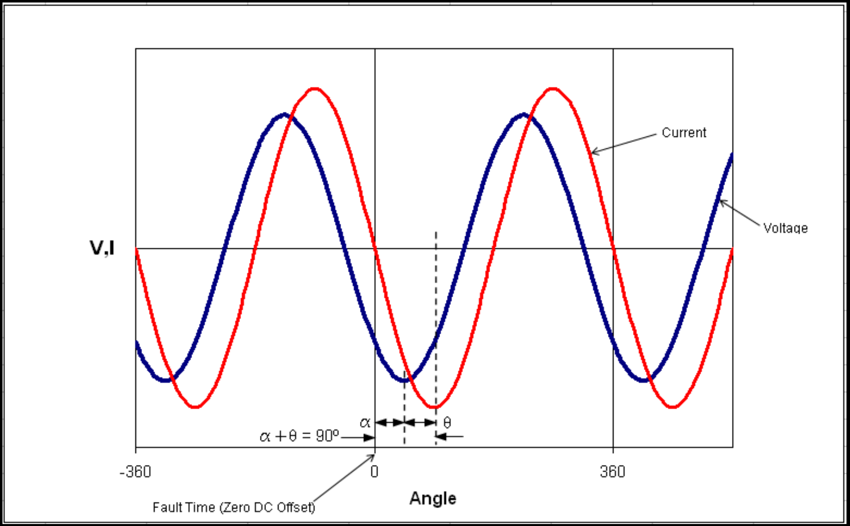
transient exponential "DC" Decay
with time constant = L/R

added to the "Steady State“ Fault Current.
lags the voltage by θ

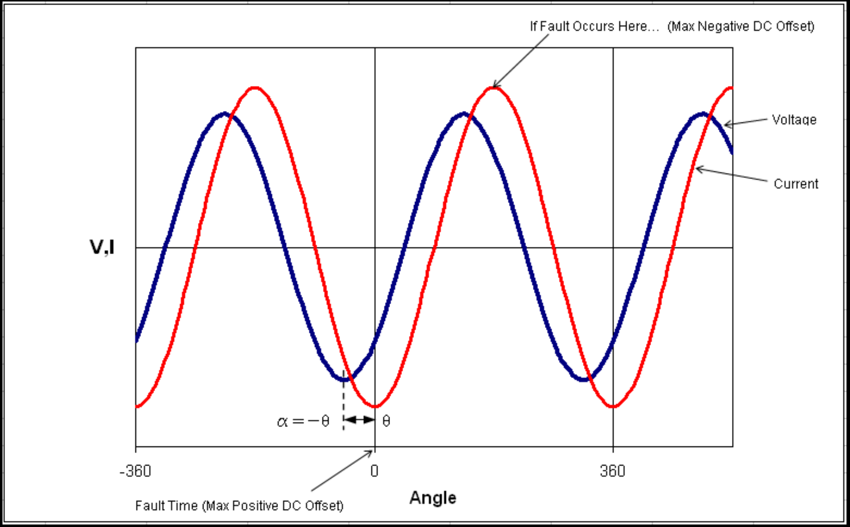
$$i(t) = \frac{V_{pk}}{|Z|}\cos(\alpha+\theta)\, e^{-\frac{R}{L}t} - \frac{V_{pk}}{|Z|}\cos(\omega t - \alpha - \theta)$$

Next question is:
what is the worst α (voltage lag) for the fault to occur?

Notice:
if $\alpha+\theta = 90°$ then $\cos(\alpha+\theta) = 0$
this would result in the DC decay portion = 0
no transient fault current.
only steady state fault current would result.
this is the "best" case scenario.



On the other hand:
if $\alpha+\theta = 0°$ then $\cos(\alpha+\theta) = 1$ (max)
this would result in the worst case DC offset scenario.
In other words…
if the fault occurs at a point in the cycle when the steady state fault current would be max ...
the worst case transient situation occurs.
Also notice:
the DC offset is max positive when the fault occurs at a negative current maximum and max negative when the fault occurs at a positive current maximum.



Therefore:
we will us $\alpha = -\theta$ and proceed

substituting $\alpha = -\theta$

$$i(t) = \frac{V_{pk}}{|Z|}\cos(\alpha + \theta)\, e^{-\frac{R}{L}t} - \frac{V_{pk}}{|Z|}\cos(\omega t - \alpha - \theta)$$

$$i(t) = \frac{V_{pk}}{|Z|}\cos(-\theta + \theta)\, e^{-\frac{R}{L}t} - \frac{V_{pk}}{|Z|}\cos(\omega t + \theta - \theta)$$

$$i(t) = \underbrace{\frac{V_{pk}}{|Z|} e^{-\frac{R}{L}t}}_{\text{"DC" offset with time constant = L/R}} - \underbrace{\frac{V_{pk}}{|Z|}\cos(\omega t)}_{\text{Peak “AC” fault current}}$$

"DC" offset with
time constant = L/R

Peak “AC” fault current

Now consider RMS values:

fault current is always considered RMS unless specified otherwise.

the RMS fault current will now be referred to as asymmetrical current = I_{asym}

recall that:

$$I_{RMS} = \sqrt{(I_{DC})^2 + \left(\frac{I_{pk}}{\sqrt{2}}\right)^2}$$

$$I_{asym} = \sqrt{\underbrace{\left(\frac{V_{pk}}{|Z|} e^{-\frac{R}{L}t}\right)^2}_{\text{"DC" offset}} + \underbrace{\left(\frac{V_{pk}}{\sqrt{2}|Z|}\right)^2}_{\text{steady state RMS fault current}}}$$

"DC" offset
still a function of time

steady state RMS fault current
referred to as symmetrical fault current = I_{sym}

$$substitute: \quad I_{sym} = \frac{V_{pk}}{\sqrt{2}|Z|}$$

$$I_{asym} = \sqrt{\left(\frac{V_{pk}}{|Z|} e^{-\frac{R}{L}t}\right)^2 + \left(\frac{V_{pk}}{\sqrt{2}|Z|}\right)^2}$$

$$I_{asym} = \sqrt{\left(\sqrt{2} I_{sym} e^{-\frac{R}{L}t}\right)^2 + \left(I_{sym}\right)^2}$$

$$I_{asym} = I_{sym}\sqrt{\left(\sqrt{2} e^{-\frac{R}{L}t}\right)^2 + 1}$$

$$I_{asym} = I_{sym}\sqrt{1 + 2e^{-\frac{2R}{L}t}}$$

$now\ convert\ time\ to\ number\ of\ cycles:$

$$n = \frac{t}{T} = ft = \frac{\omega t}{2\pi}$$

$$t = \frac{2\pi n}{\omega}$$

$$substitute: \;\; t = \frac{2\pi n}{\omega}$$

$$I_{asym} = I_{sym}\sqrt{1 + 2e^{\frac{-2R}{L}t}}$$

$$I_{asym} = I_{sym}\sqrt{1 + 2e^{\frac{-2R2\pi n}{\omega L}}}$$

$$I_{asym} = I_{sym}\sqrt{1 + 2e^{\frac{-4\pi nR}{\omega L}}}$$

$$I_{asym} = I_{sym}\underbrace{\sqrt{1 + 2e^{\frac{-4\pi n}{X/R}}}}$$

instantaneous fault current now a function of cycles

the RMS fault current decays to the steady state or symmetrical fault current…
how many cycles it takes depends on the X/R Ratio.
for “typical" X/R ratio of 17… the time constant is 2.7 cycles.

Now consider how breakers are affected by this transient fault current:

The IEEE standard that specifies the interrupting capability of breakers states that the breaker shall be capable of interrupting the asymmetrical fault current.

This derivation shows that the asymmetrical fault current that the breaker must interrupt depends on how long it takes the breaker to clear the fault.

<u>In other words:</u>
if a breaker has a 20kA rating...
this is a symmetrical or steady state current rating.
the breaker must be capable of interrupting more than 20kA...
again, how much more depends on how quickly it reacts to a fault.
the faster it reacts, the more fault current it must clear.

The IEEE standard uses a typical system X/R ratio = 17 when specifying breaker interrupting capability.

Using X/R =17…
The plot below shows the worst case instantaneous fault current as a function of cycles

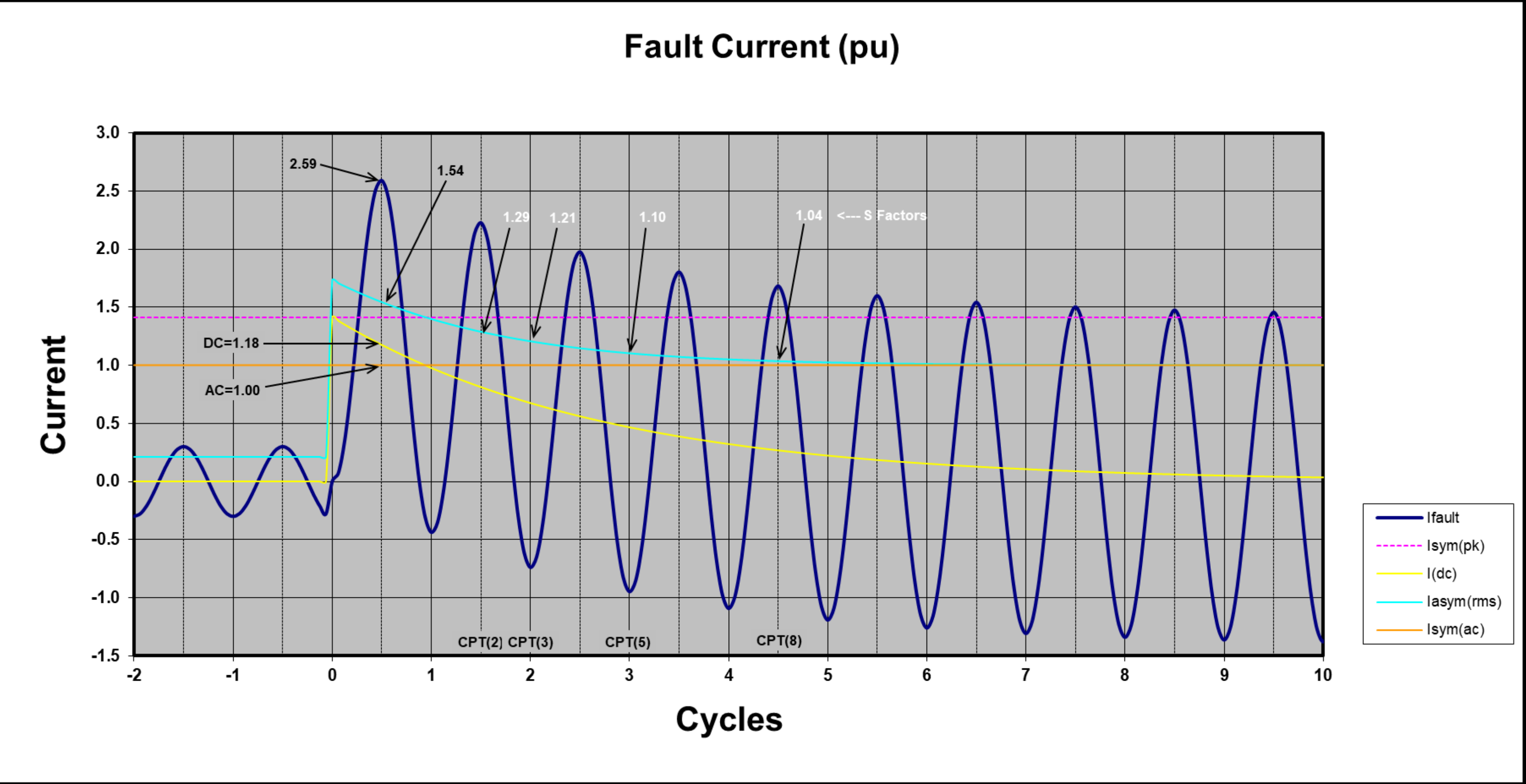
Instantaneous fault current:

$$i_F(n) = \frac{V_{pk}}{|Z|} e^{\frac{-2\pi n}{17}} - \frac{V_{pk}}{|Z|} \cos(2\pi n)$$

also shown on the plot:

$$I_{asym} = I_{sym}\sqrt{1 + 2e^{\frac{-4\pi n}{17}}}$$

$$I_{dc} = \sqrt{2} I_{sym} e^{\frac{-2\pi n}{17}}$$



contact parting time (CPT) = ½ + ½rated

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Questions or Comments …

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